

Surface Reconstruction Using Bivariate Simplex Splines on Delaunay Configurations

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Zhejiang University

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Guozhao Wang

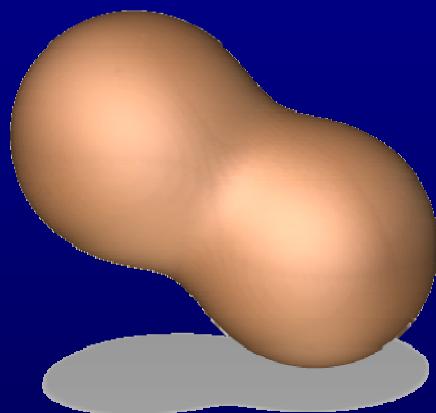
Zhejiang University

Hong Qin

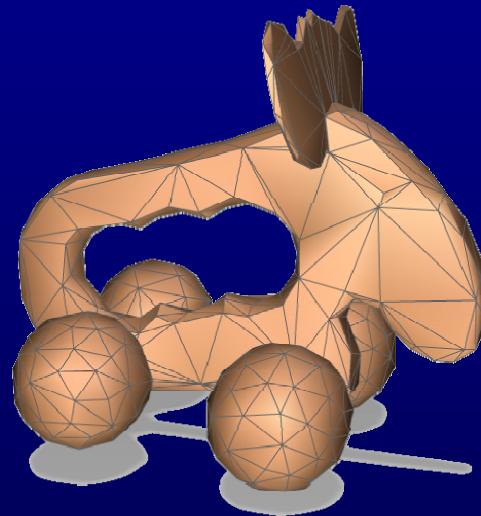
Stony Brook University

Surface Reconstruction

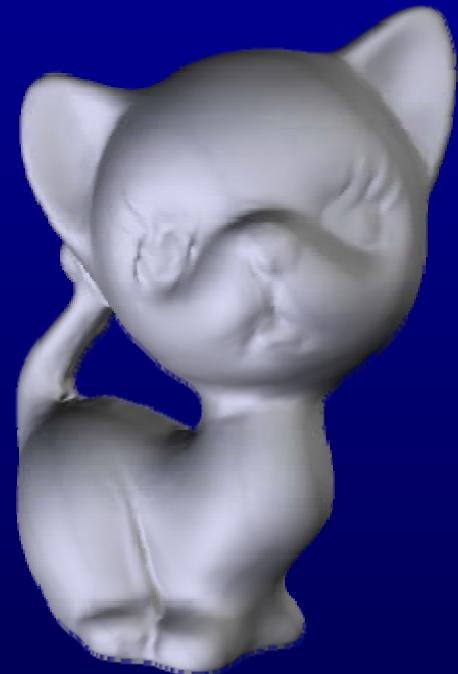
- Smooth surfaces reconstructed using different modeling representations and techniques



Algebraic surface



Subdivision surface



Spline surface

Bivariate Surfaces

- Tensor-product spline surface
 - Limitation: inherent rectilinear shape
- Non-tensor-product spline surface
 - Defined over triangular domain, multi-sided domain or scattered point set

Bivariate Surfaces

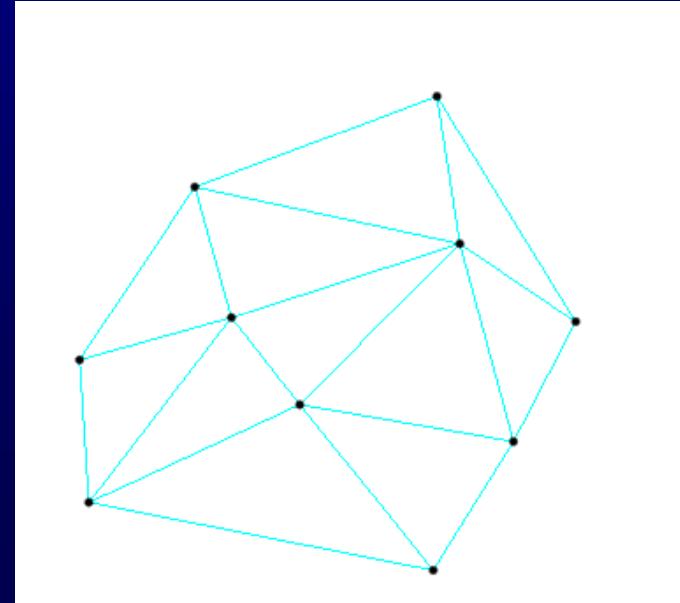
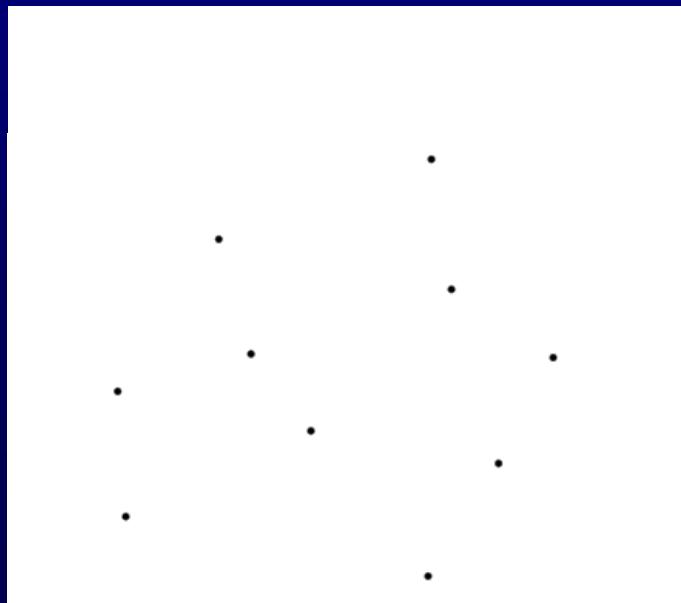
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Bivariate Simplex Splines on Delaunay Configurations

- Bivariate simplex spline [de Boor 72]
 - Properties similar to individual univariate B-spline basis
 - Can be recursively calculated

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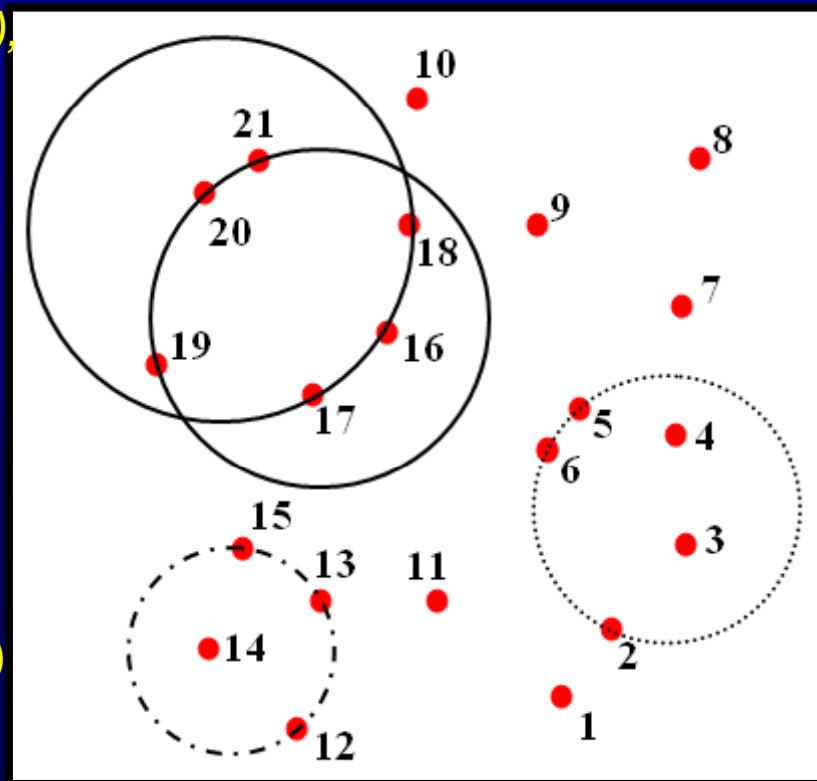
- Delaunay Configurations (DC)

(B, I) : the circle through B only containing I inside

Degree-3 DC:

$(\{16, 17, 18\}, \{19, 20, 21\})$,
 $(\{19, 20, 21\}, \{16, 17, 18\})$

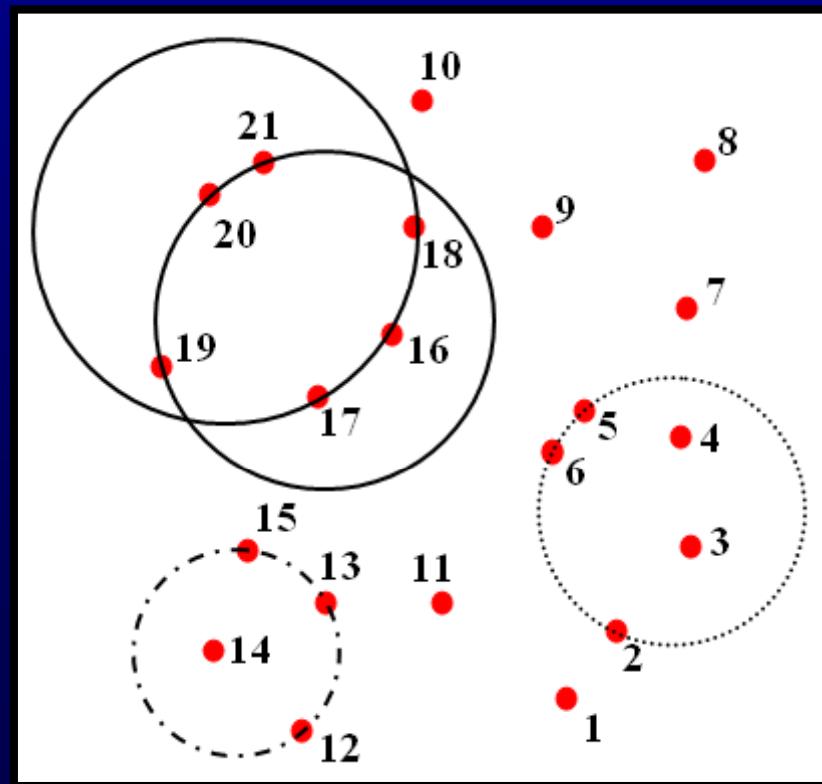
Degree-1 DC
 $(\{12, 13, 15\}, \{14\})$



Degree-2 DC
 $(\{2, 5, 6\}, \{3, 4\})$

Bivariate Simplex Splines on Delaunay Configurations

- Bivariate splines on Delaunay Configurations
 - Simplex space



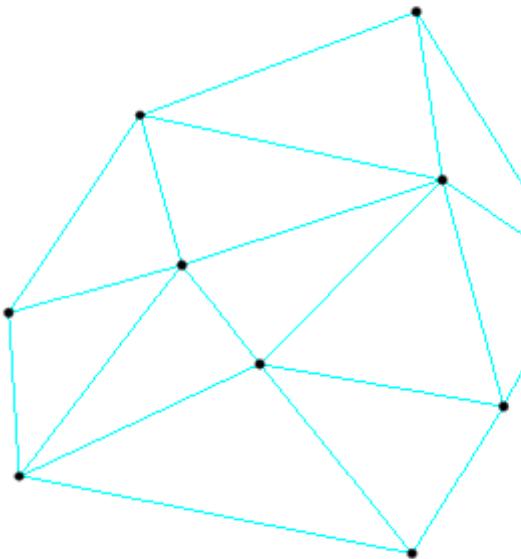
Major Contributions

- No auxiliary knots
- Handle triangular mesh
- Adaptive surface refining

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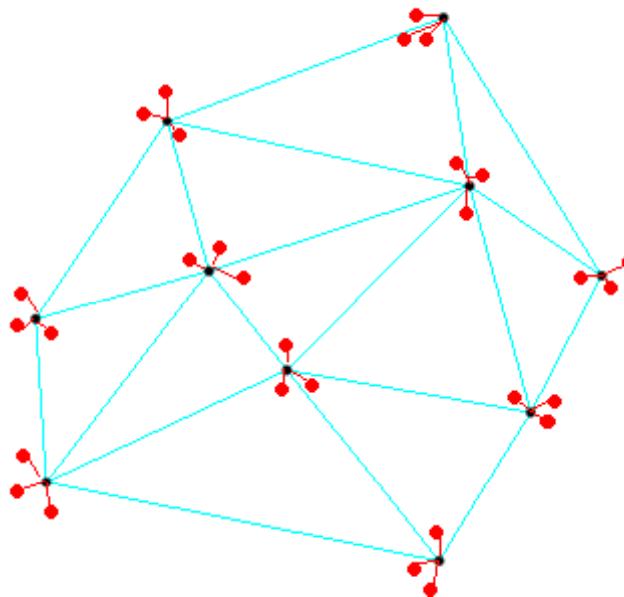
DMS [Dahmen et al 1992]



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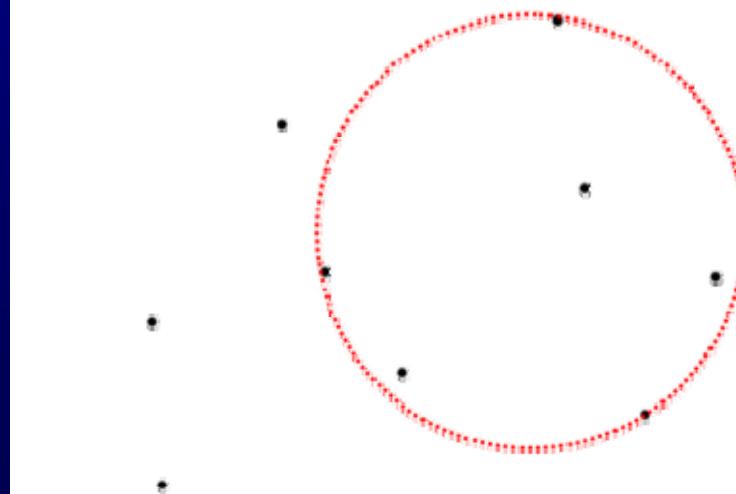
DMS: auxiliary knots are needed



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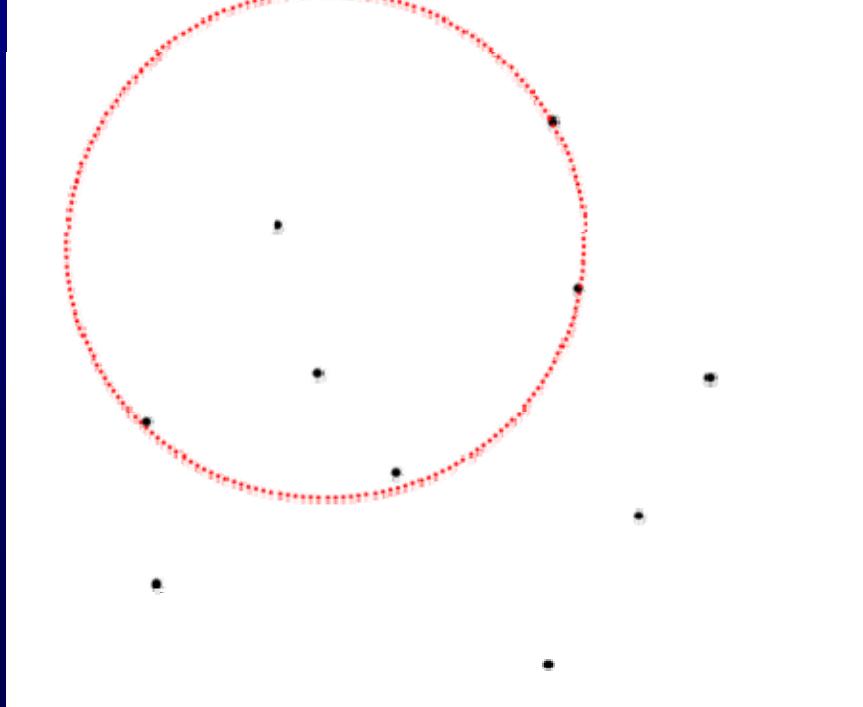
DCB: use circle to choose knots



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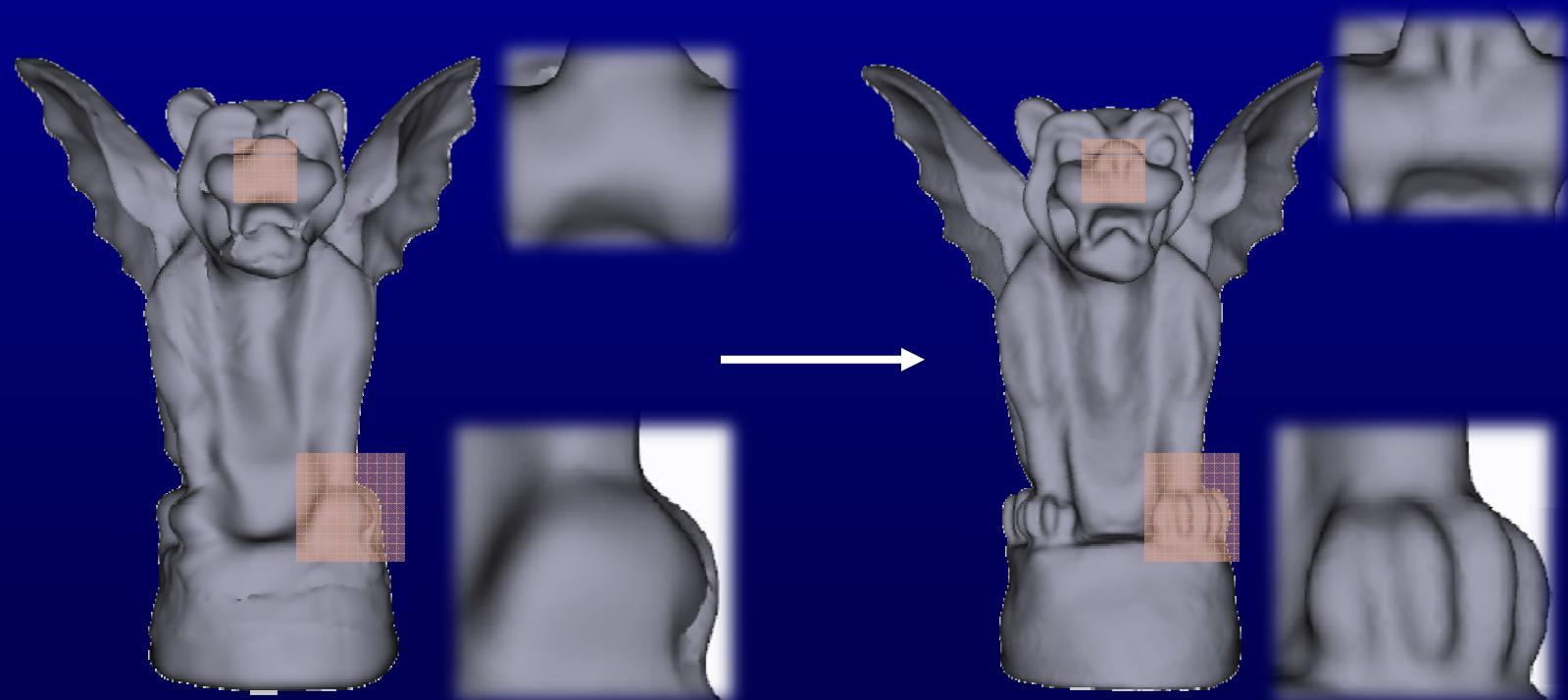


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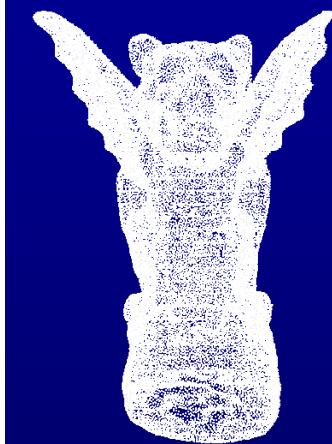
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Smooth surface at a coarse scale Refined surface

Overview



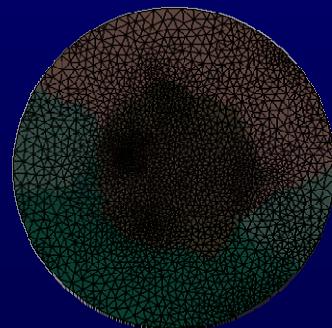
Original data



Segmentation

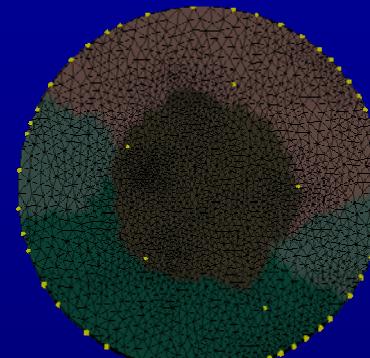


Expansion

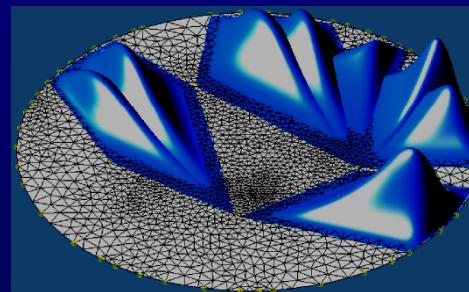


Parameterization

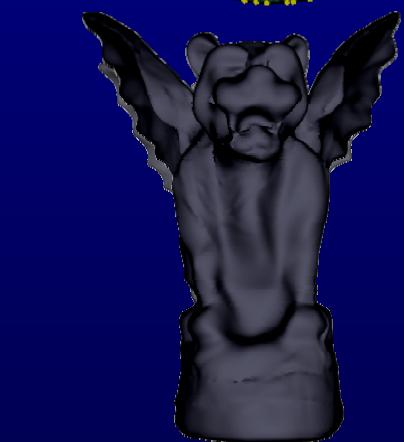
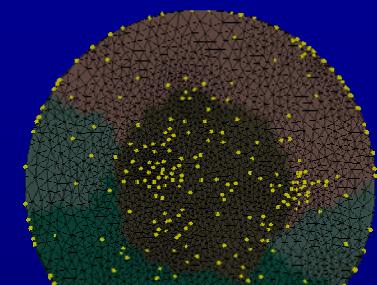
Chart Generating



Knot placement

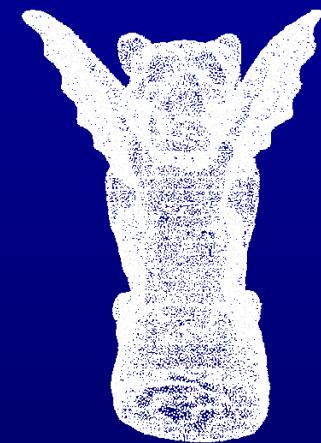


Blending



Surface Refining

Overview



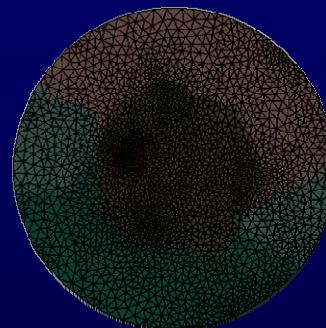
Original data



Segmentation

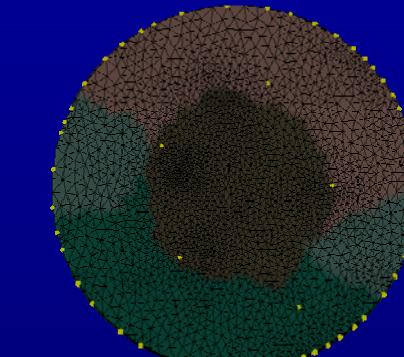


Expansion

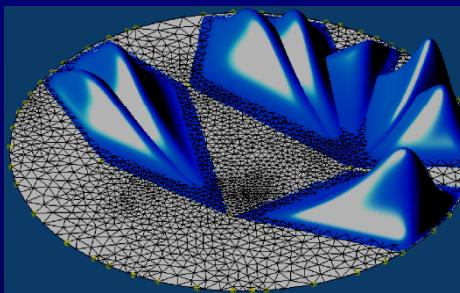


Parameterization

Chart Generating

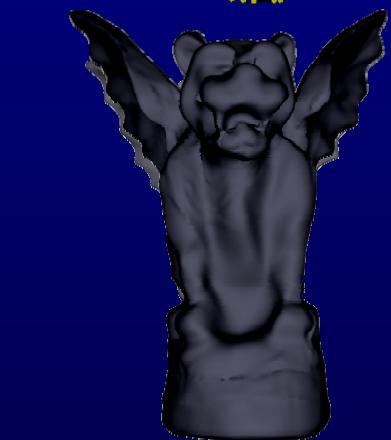
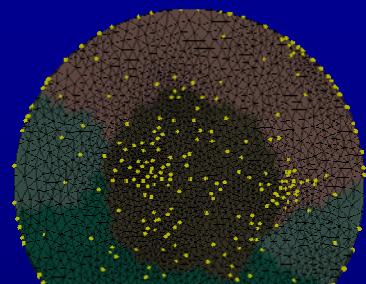


Knot placement



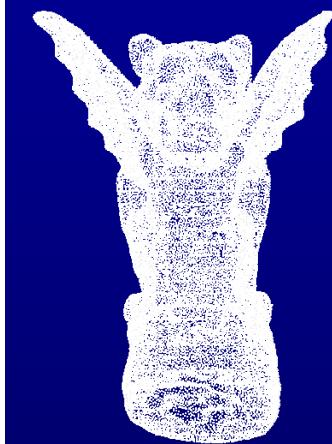
Blending

Patch Blending



Surface Refining

Overview



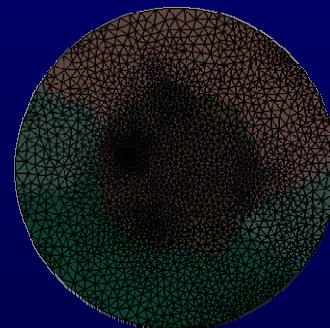
Original data



Segmentation

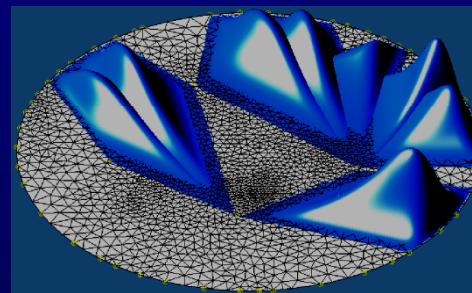


Expansion



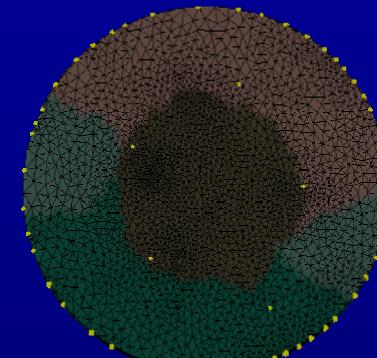
Parameterization

Chart Generating

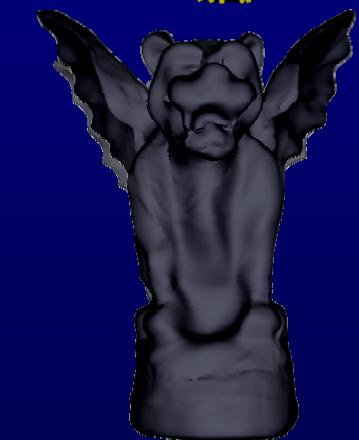
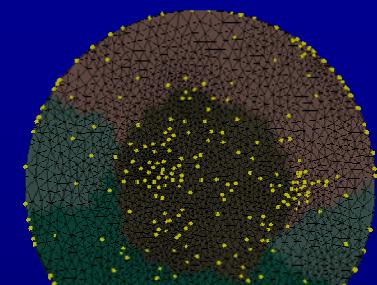


Blending

Patch Blending

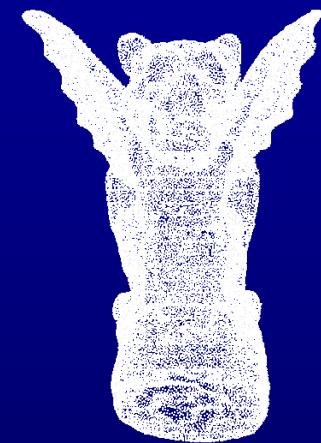


Knot placement



Surface Refining

Overview



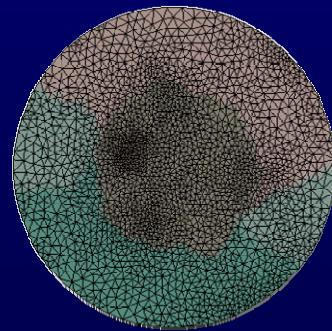
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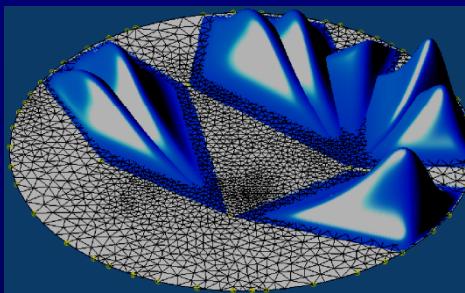


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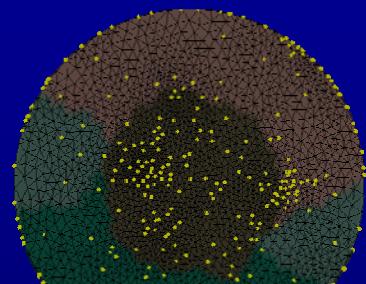
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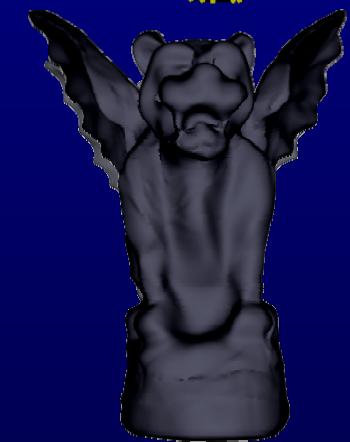


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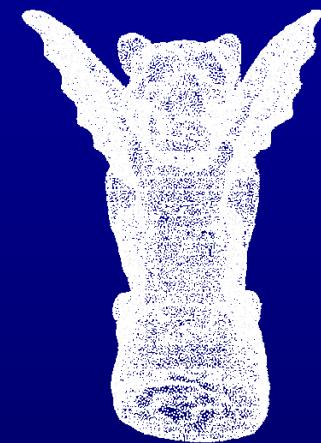


Knot placement



Surface Refining

Overview



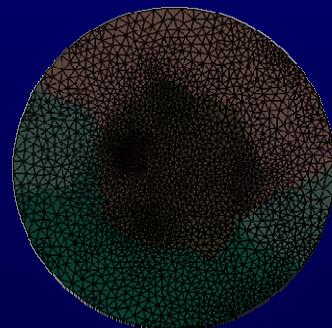
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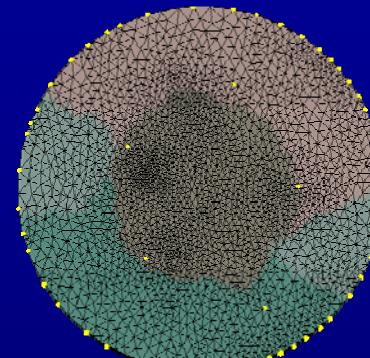


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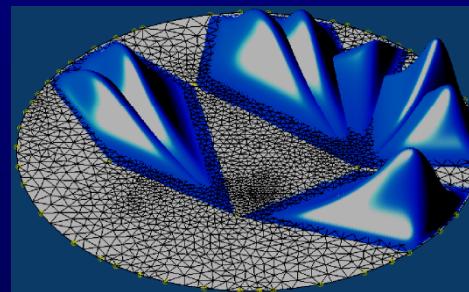


Parameterization

Chart Generating

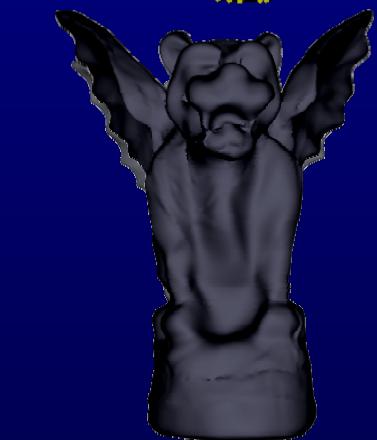
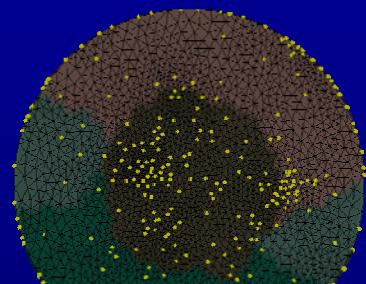


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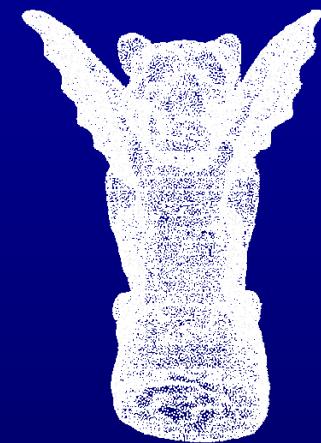
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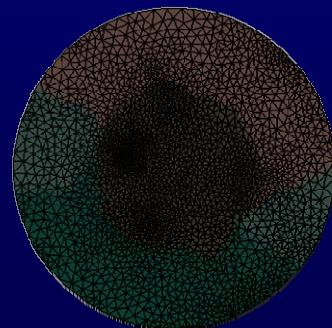
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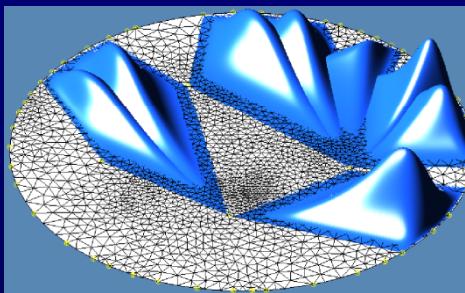


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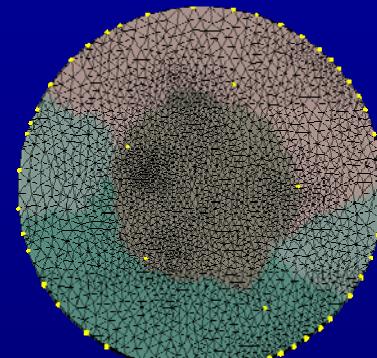
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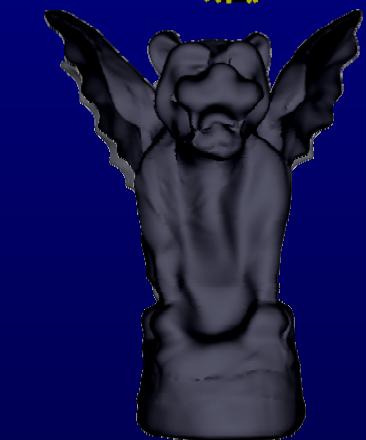
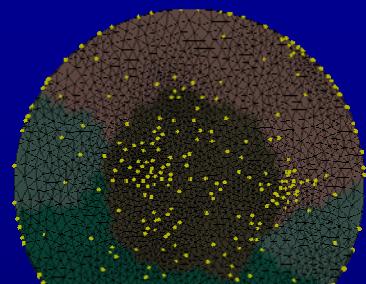


Blending

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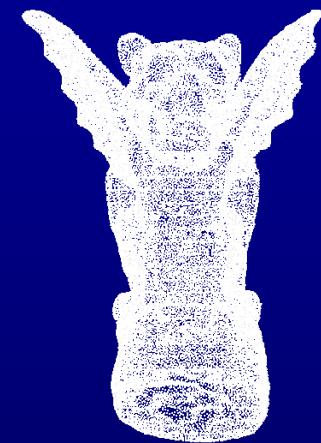


Knot placement



Surface Refining

Overview



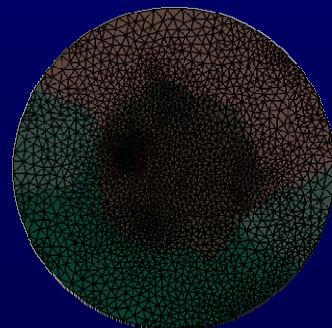
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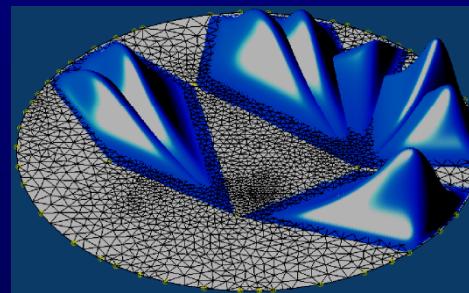


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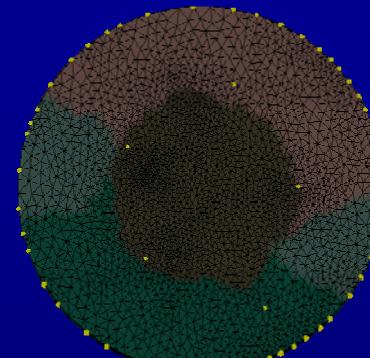
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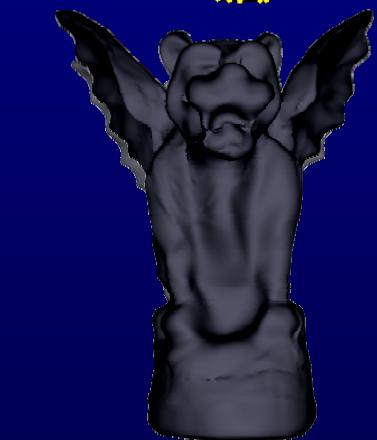
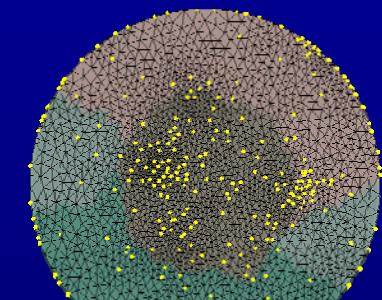


Blending

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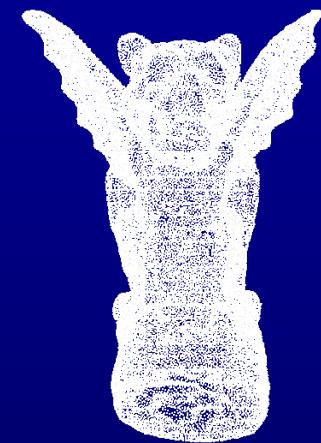


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Surface Refining

Overview



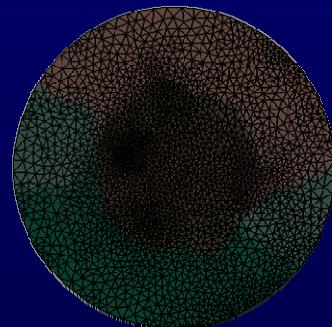
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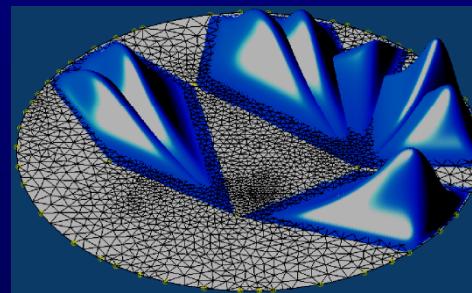


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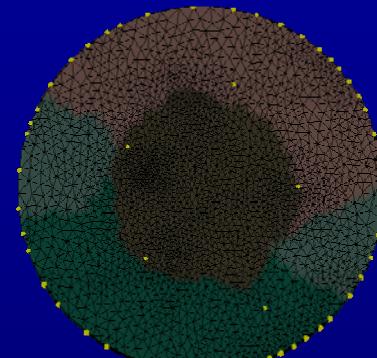
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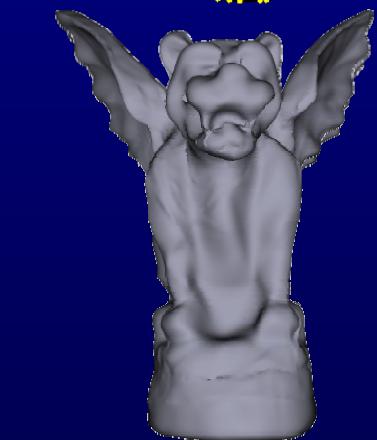
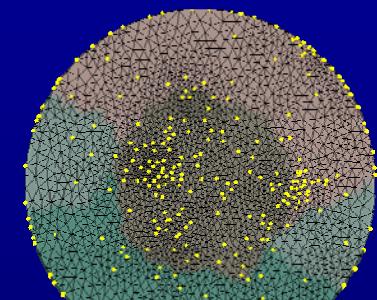


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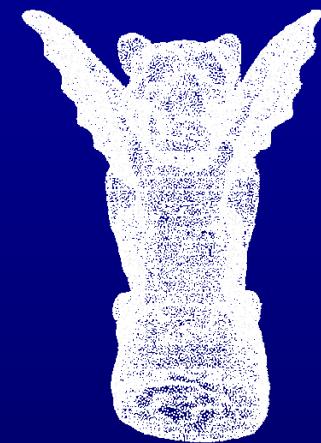


Knot placement



Surface Refining

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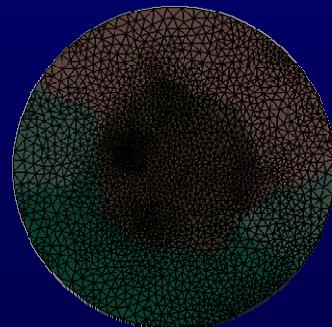
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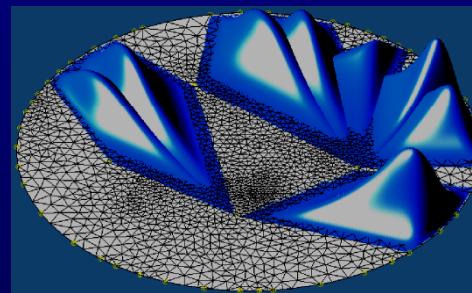


Extension



Parameterization

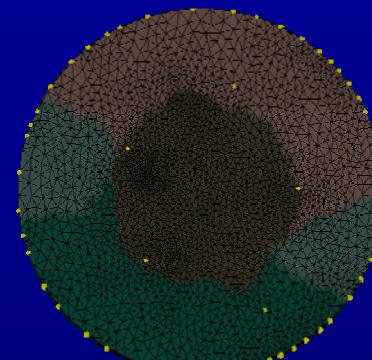
Chart Generating



Blending

Patch Blending

Surface Refining



Knot placement

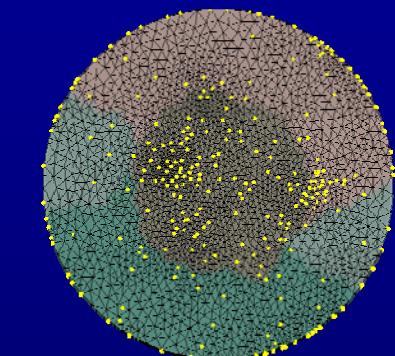
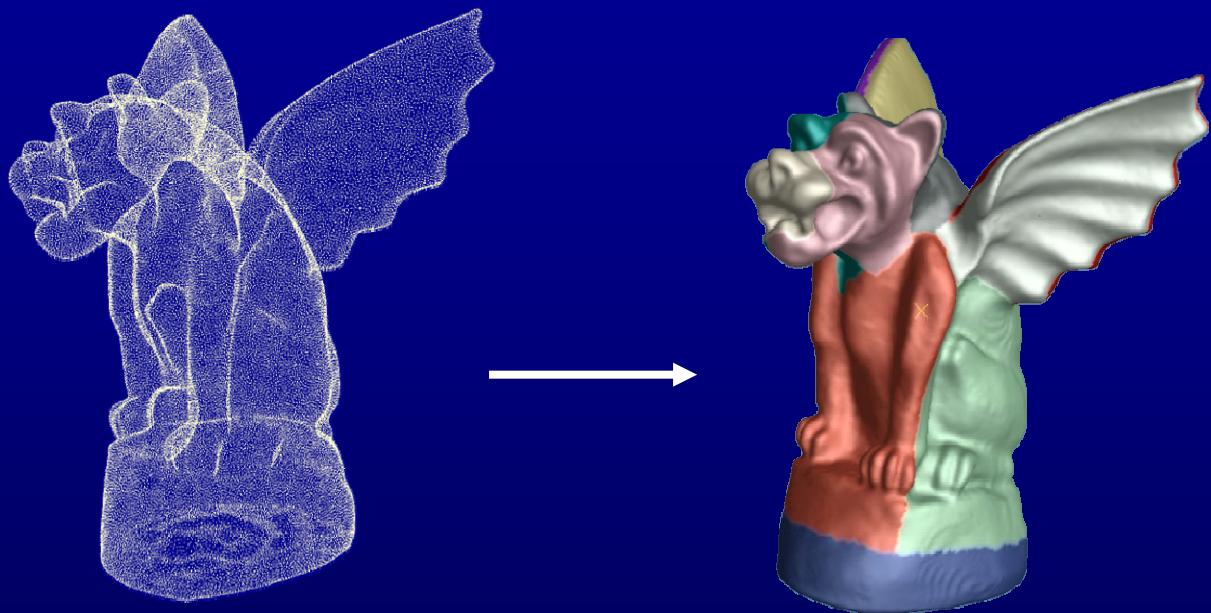
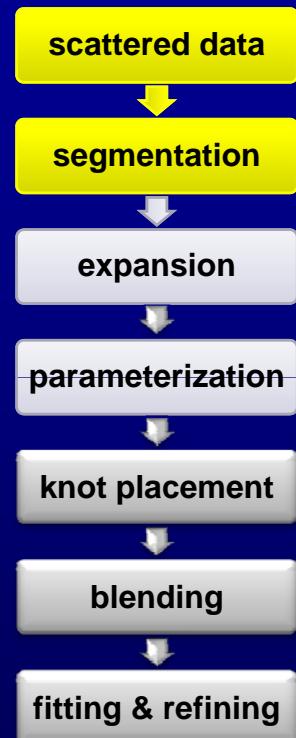
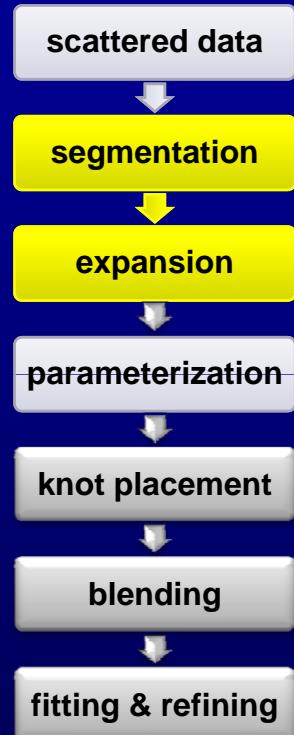


Chart Generating



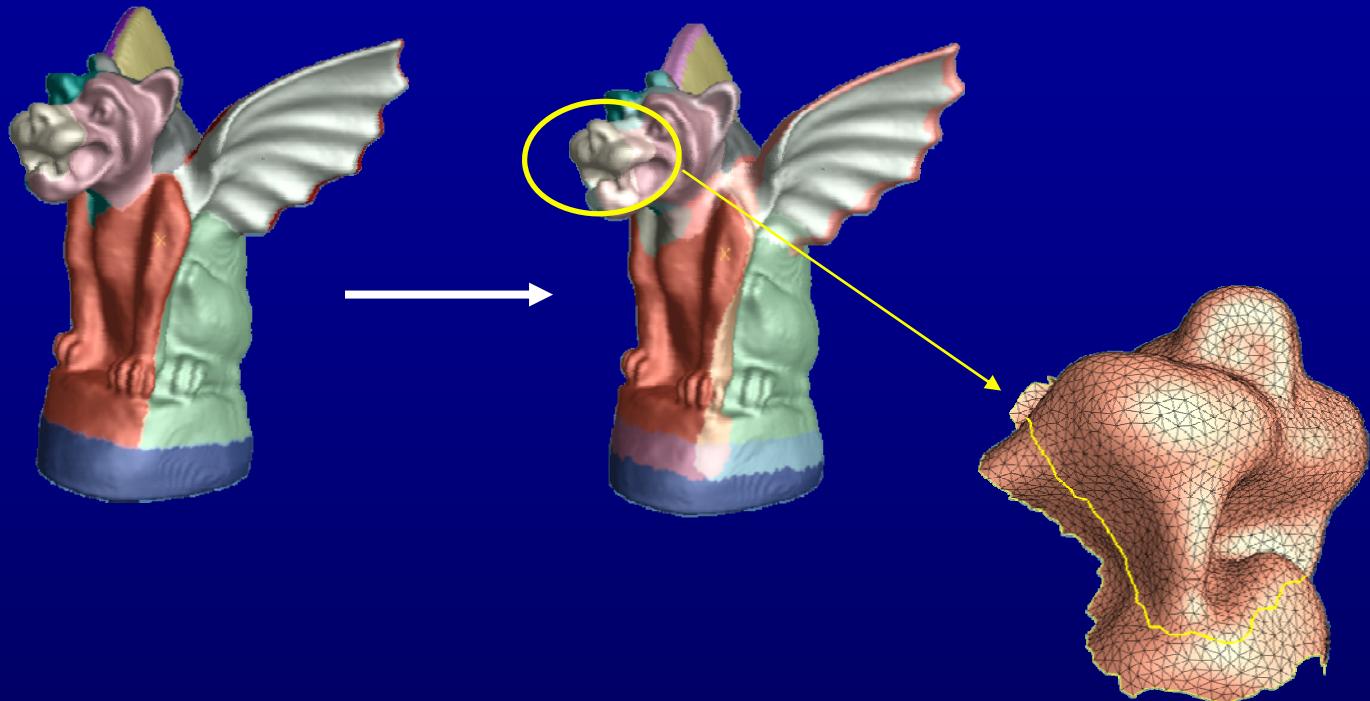
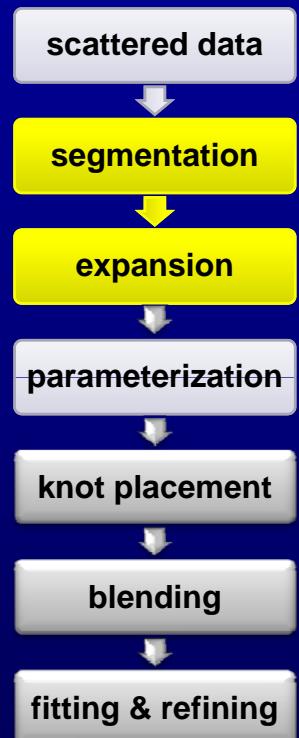
- Partition data to topological disk patches
[P. Sander et al 2003]

Chart Generating



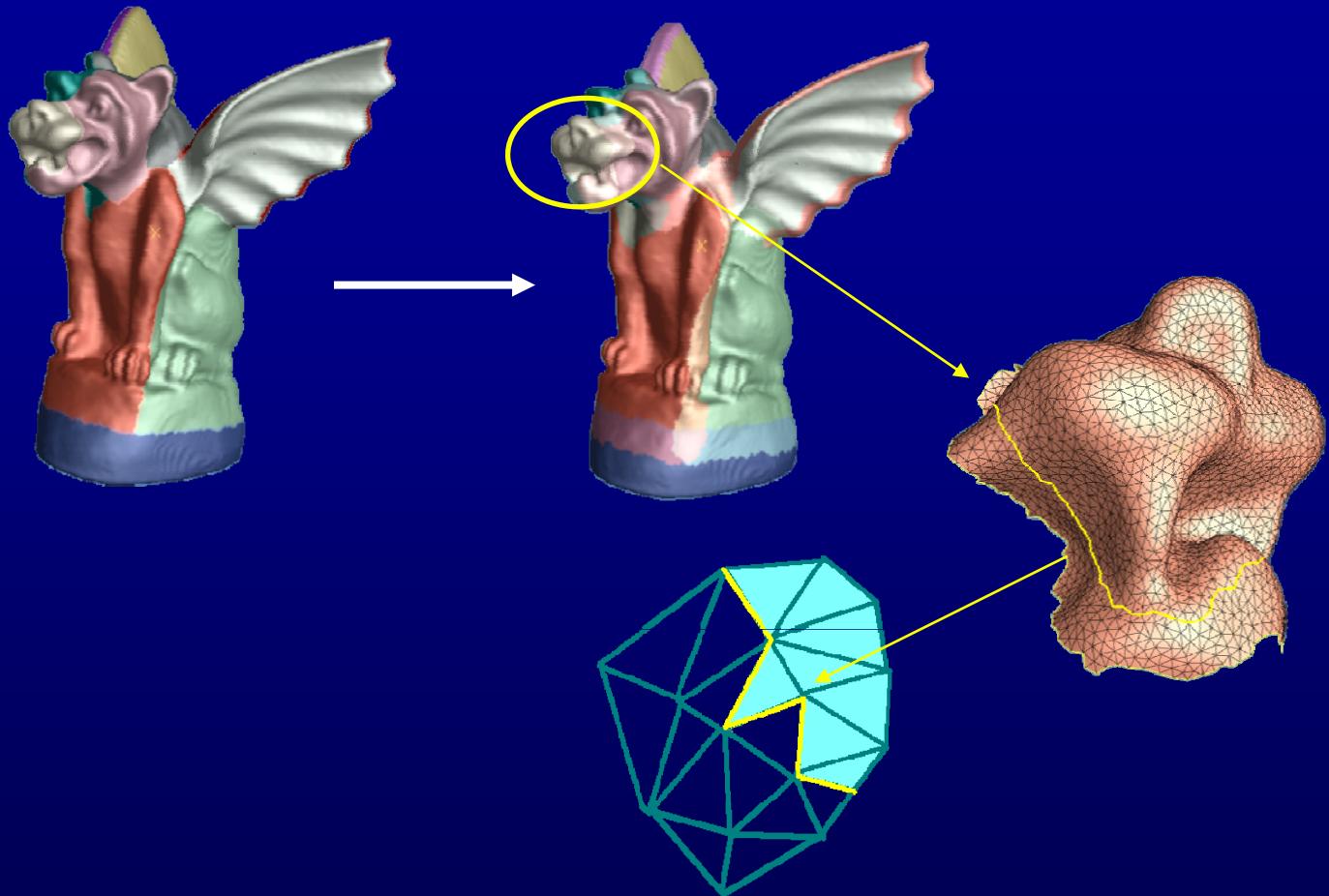
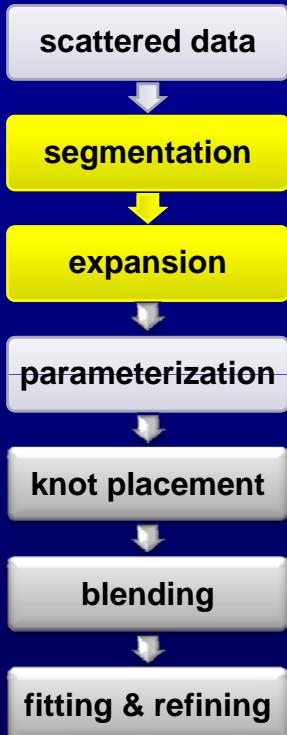
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Chart Generating



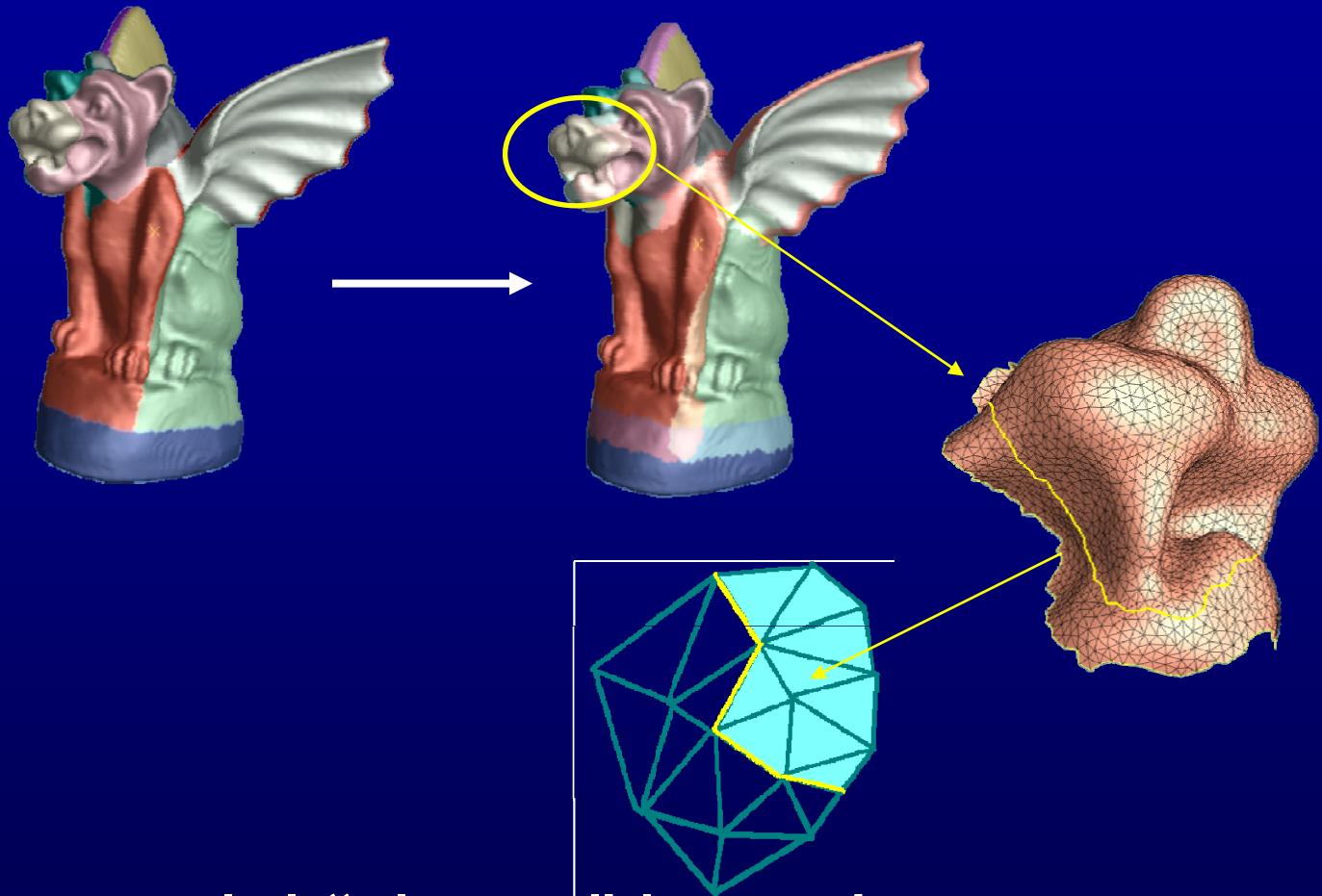
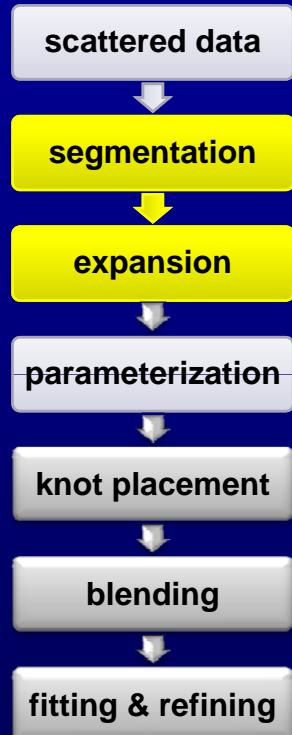
- Expand each patch outward for several rings

Chart Generating



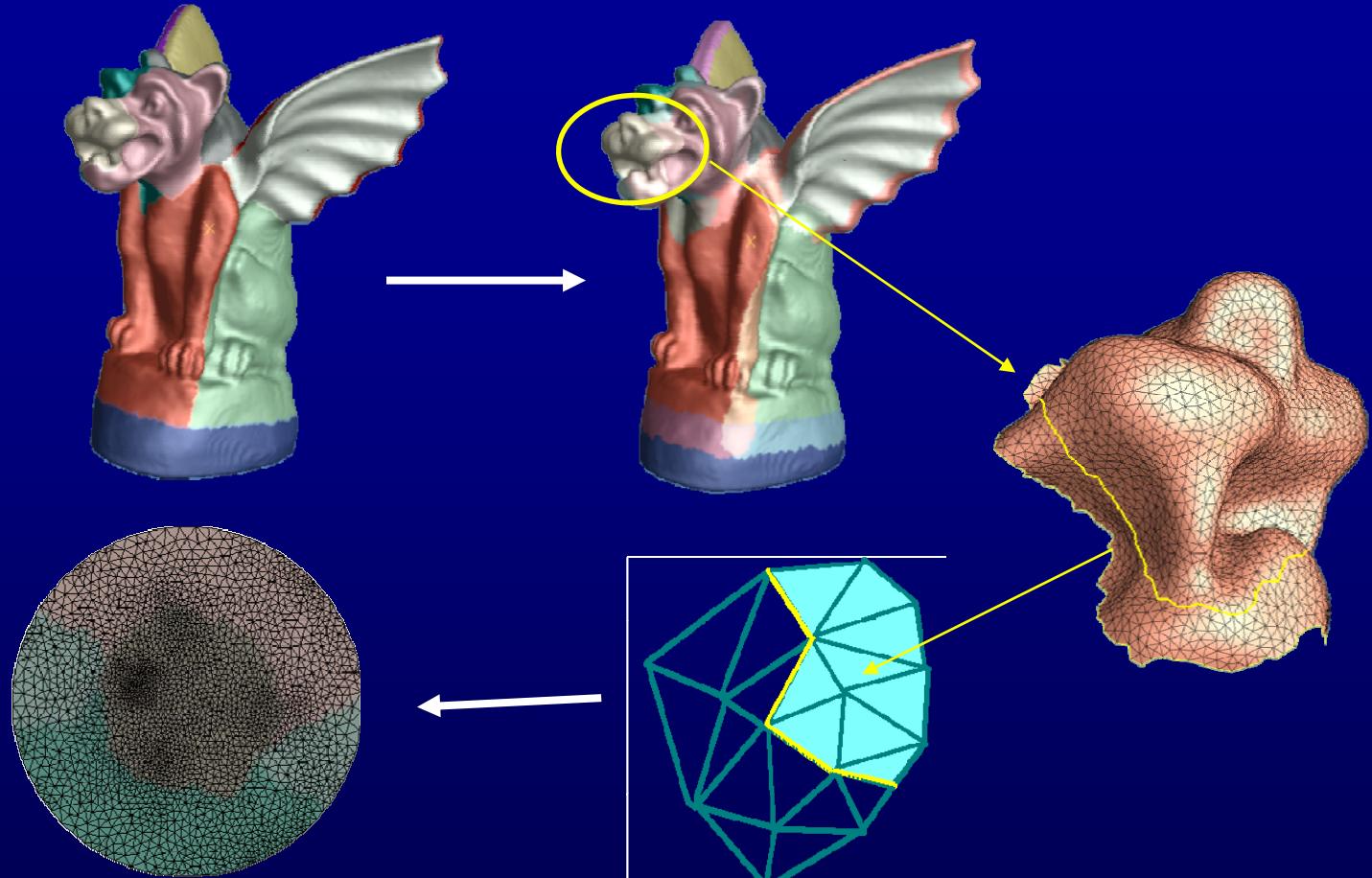
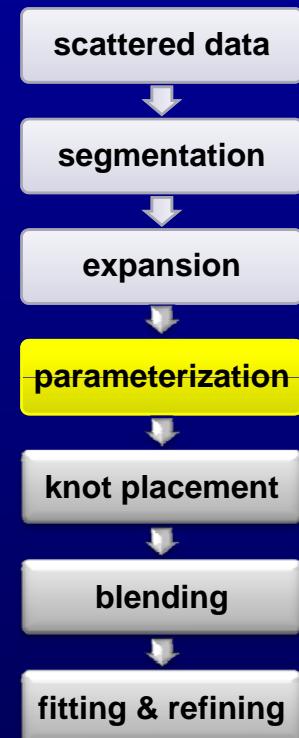
- Repair potential “zigzag” boundary

Chart Generating



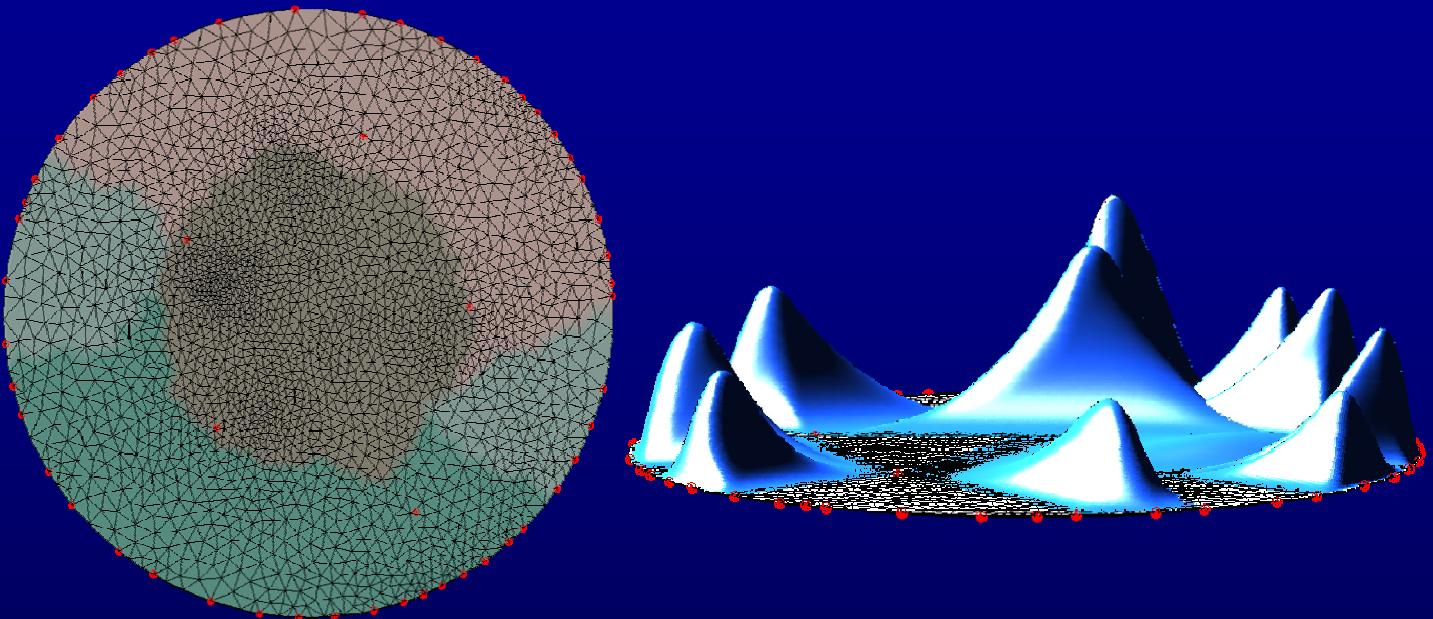
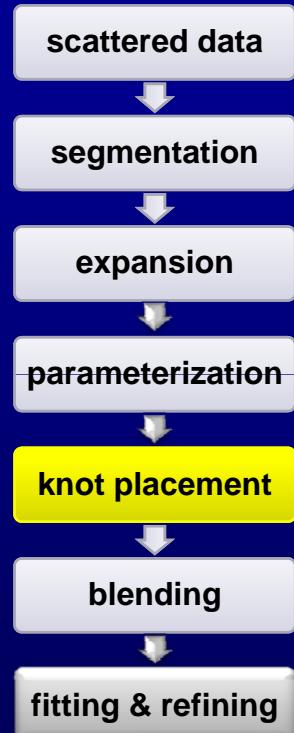
- Repair potential “zigzag” boundary

Chart Generating



- Conformal parameterization method [Li et al 2006]

Patch Blending



- Use k-mean cluster to initialize knots

Patch Blending

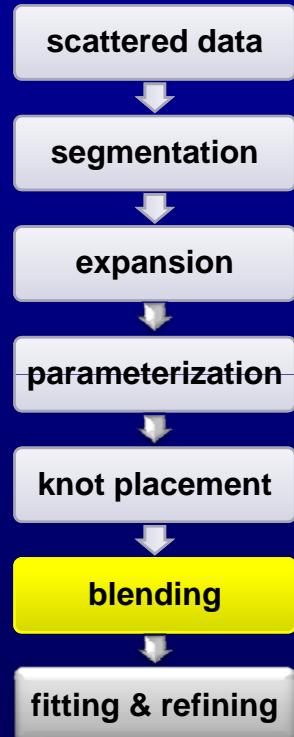
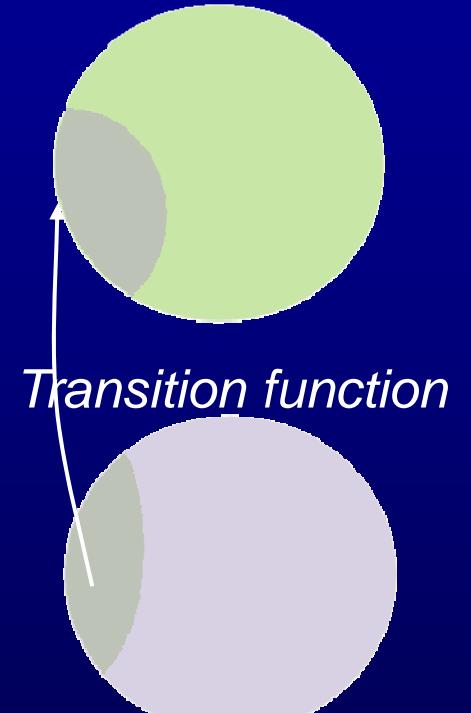


Chart 2

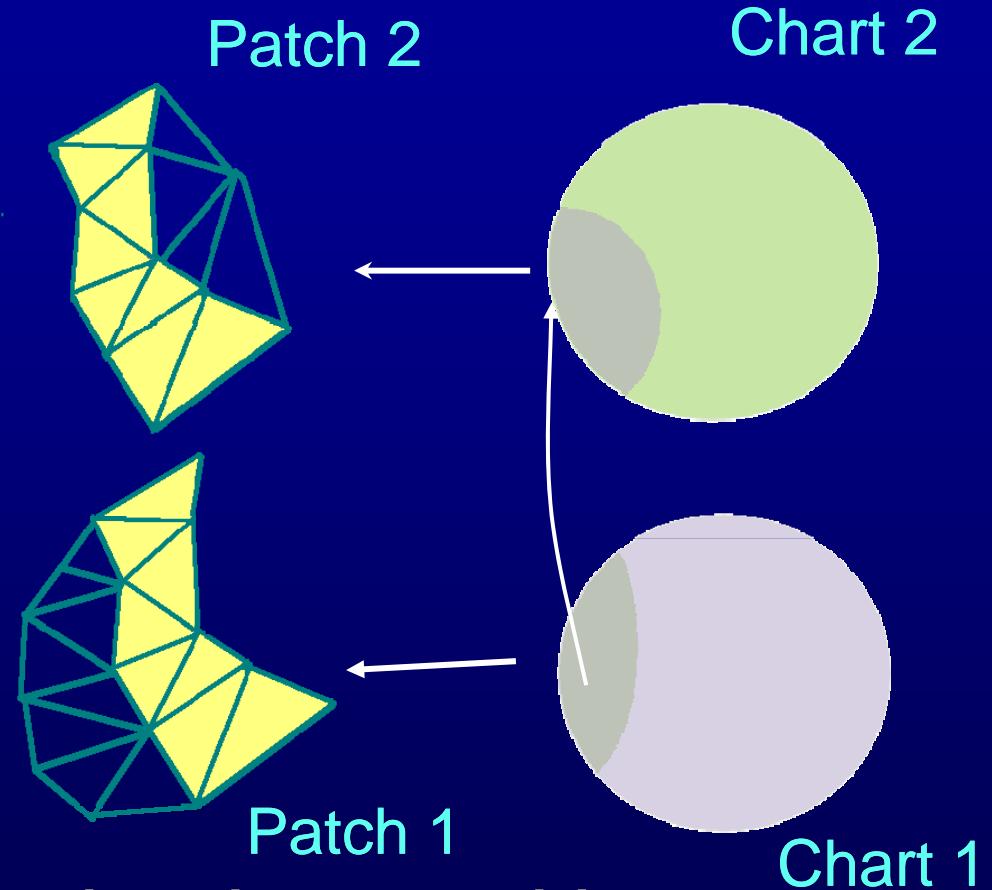
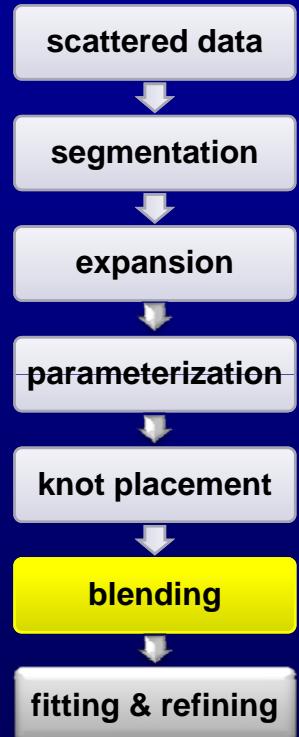


Transition function

Chart 1

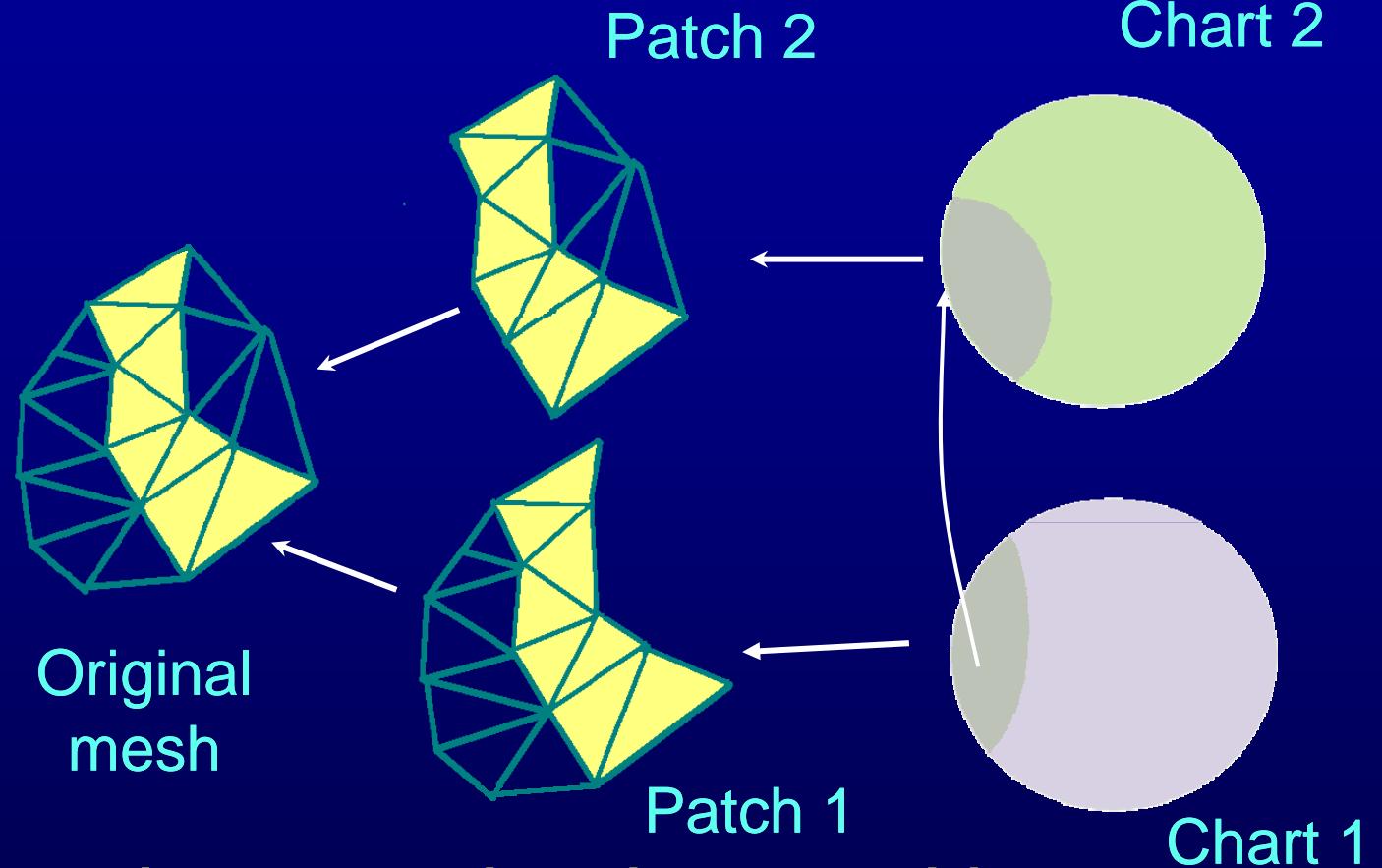
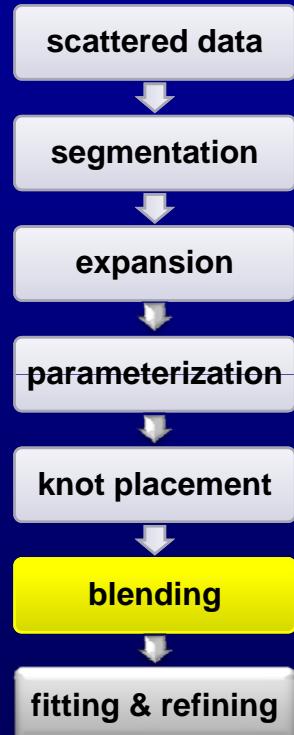
- Glue patches together by transition functions

Patch Blending



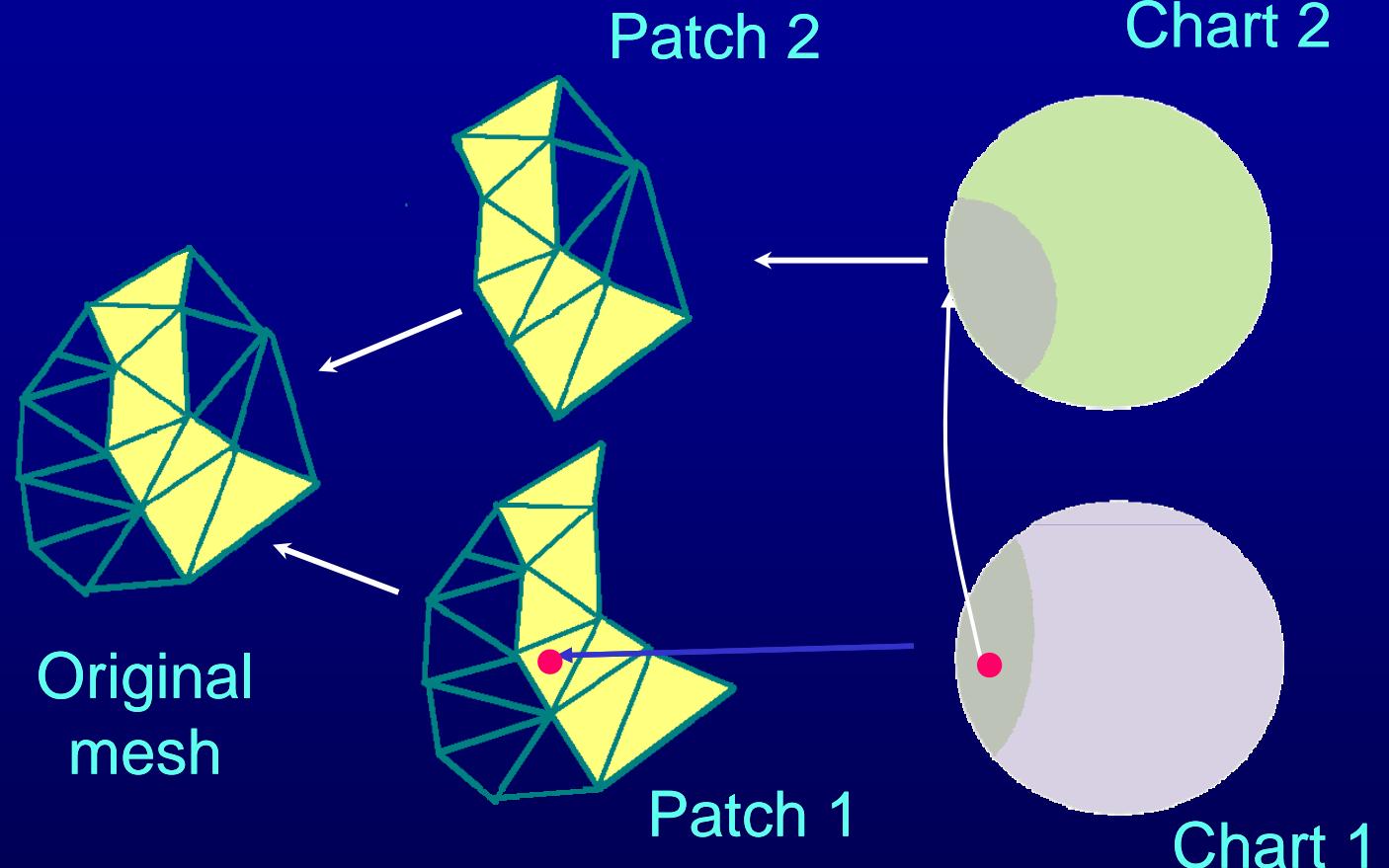
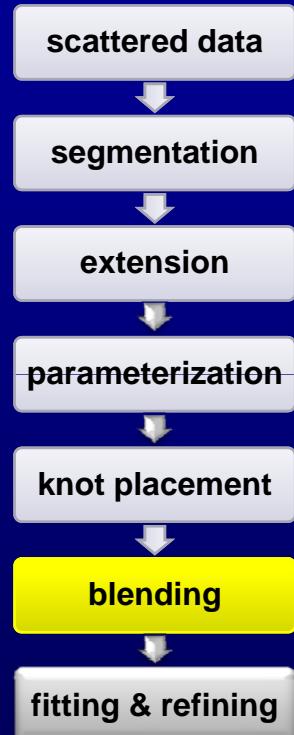
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Patch Blending



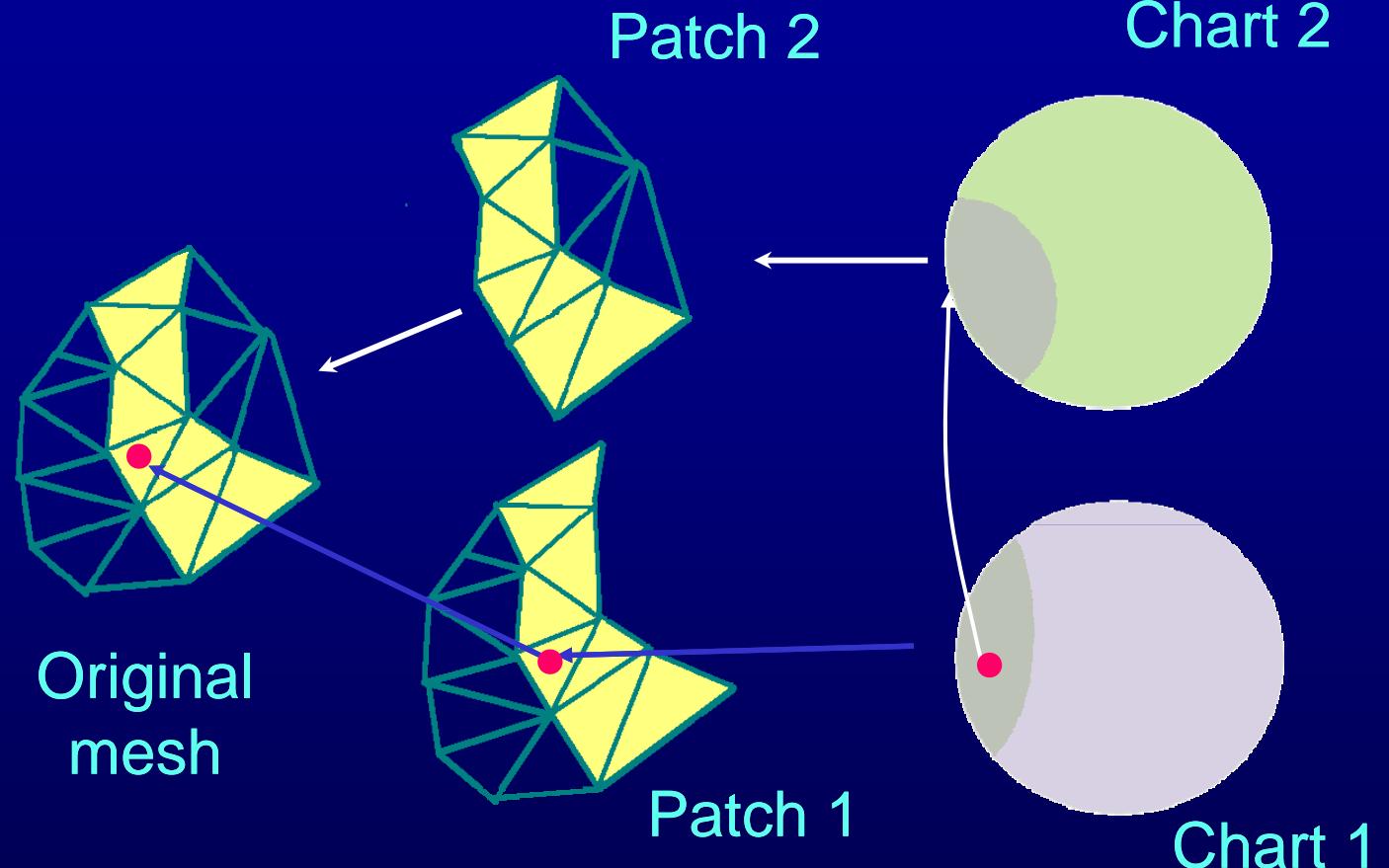
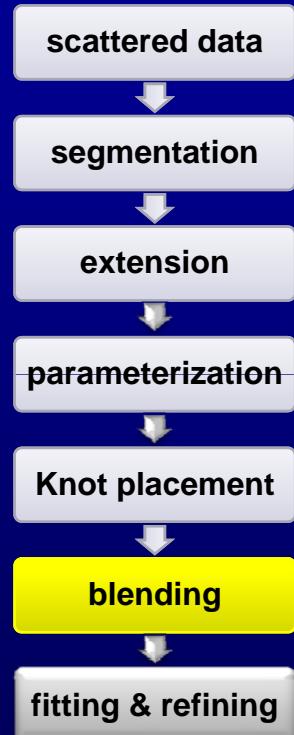
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Patch Blending



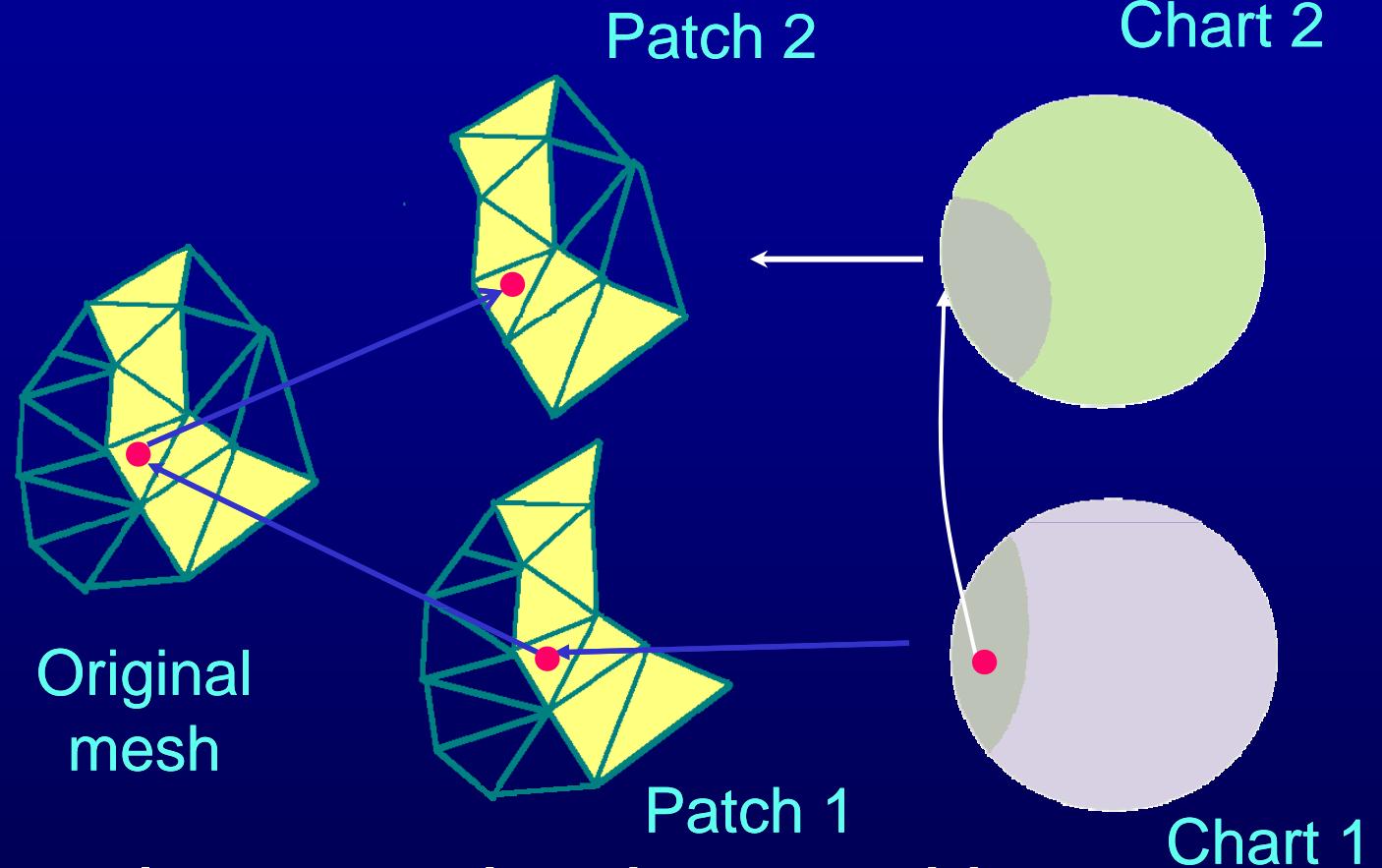
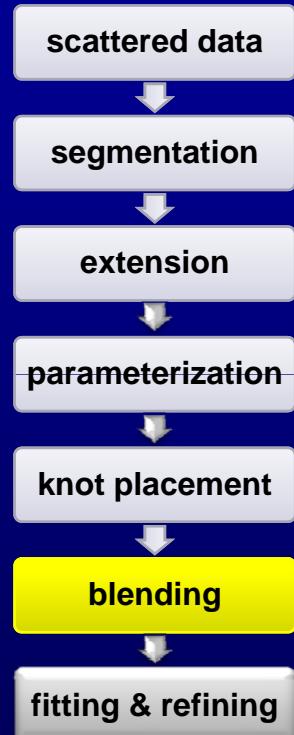
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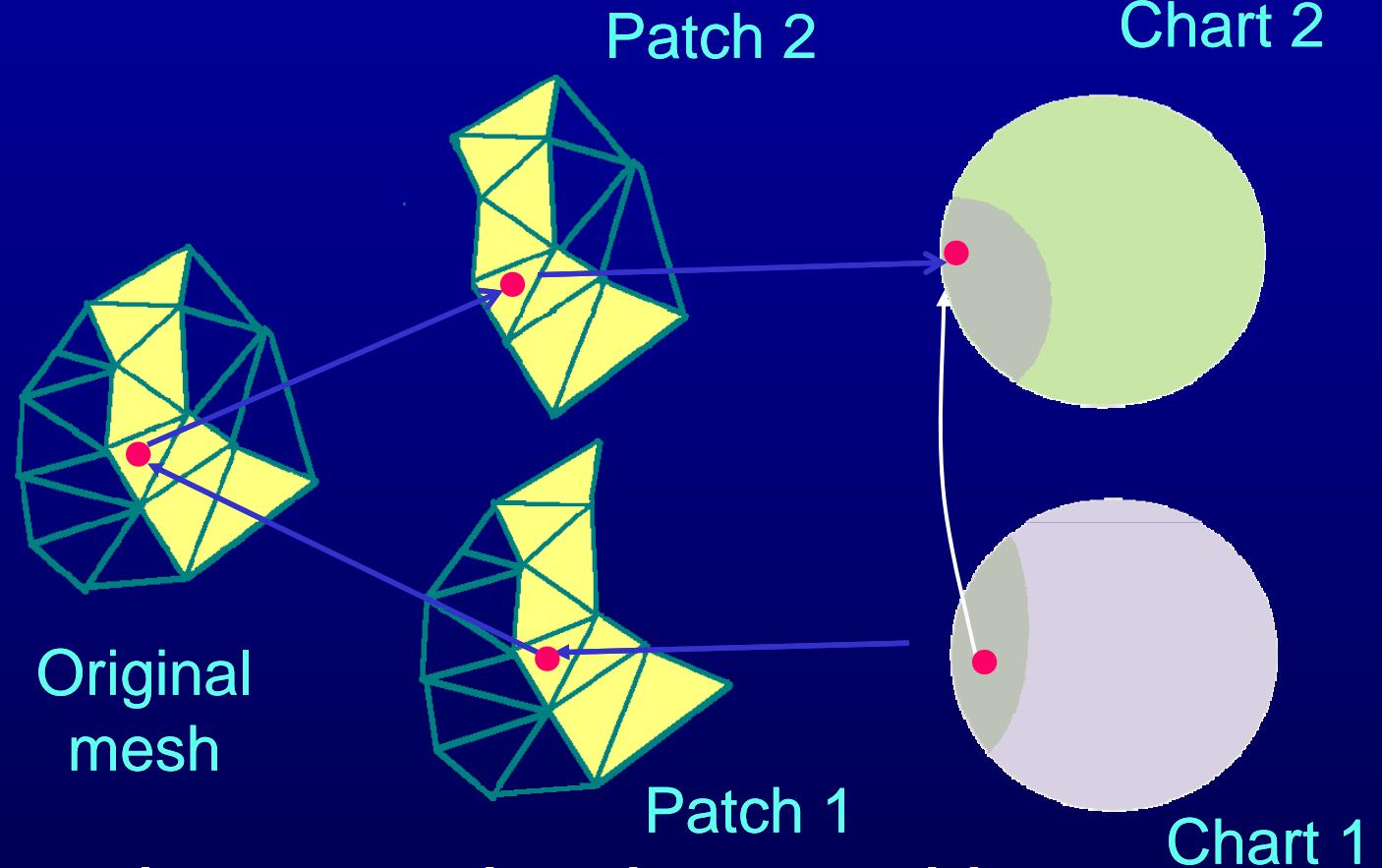
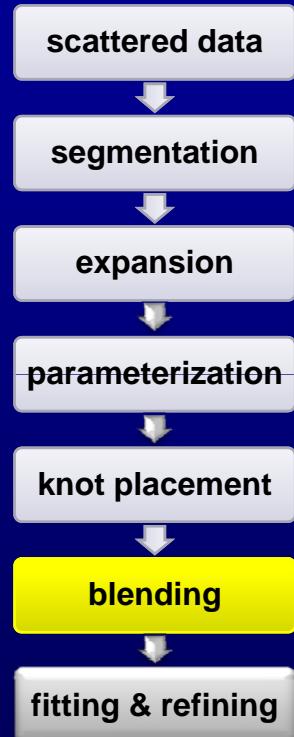
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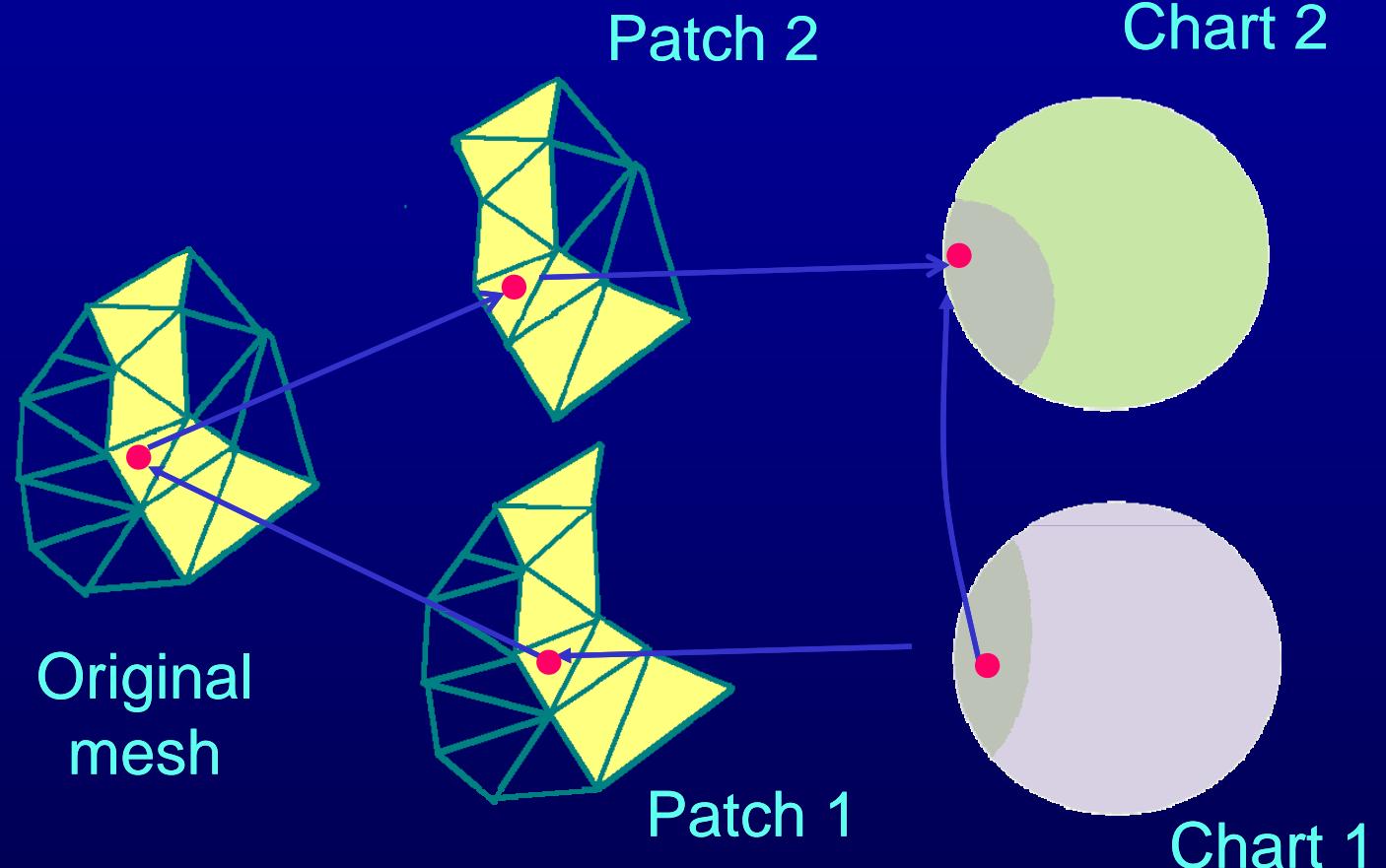
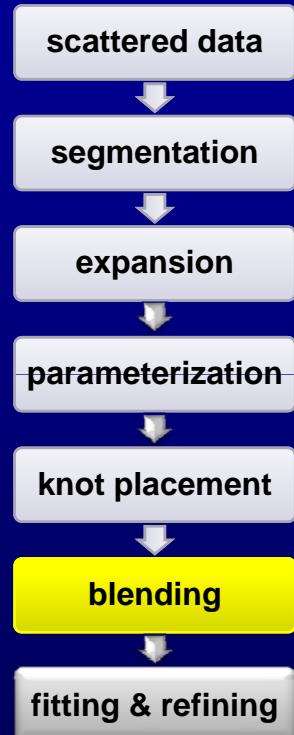
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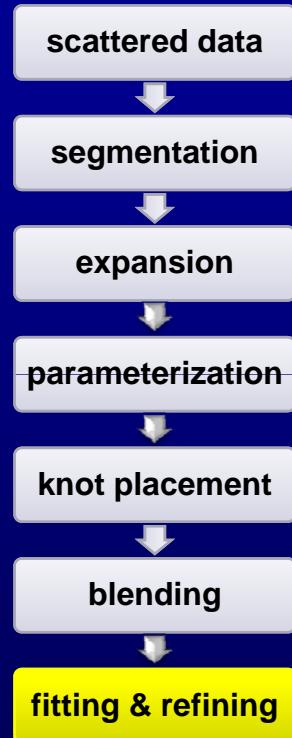
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Patch Blending



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Surface Refining

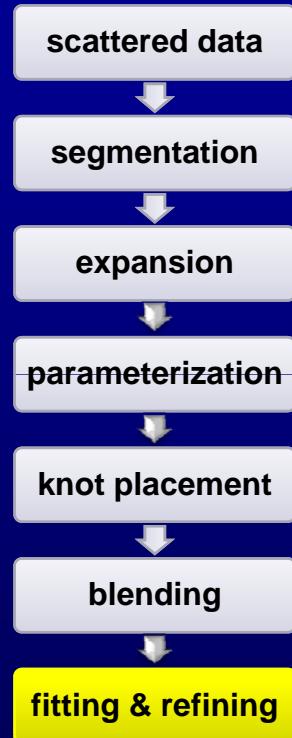


$$\text{minimize} \sum_{i=1}^n \|x_i - \sum_{j=1}^m \sum_{s=1}^{m_j} B_s^j(p_i^j) c_s^j\|^2$$

$$\text{minimize} \|X - AC\|^2$$

- Solve linear least squares problem (SVD method)
- Add new knots adaptively
- Convert to several smaller-size LLS problem

Surface Refining

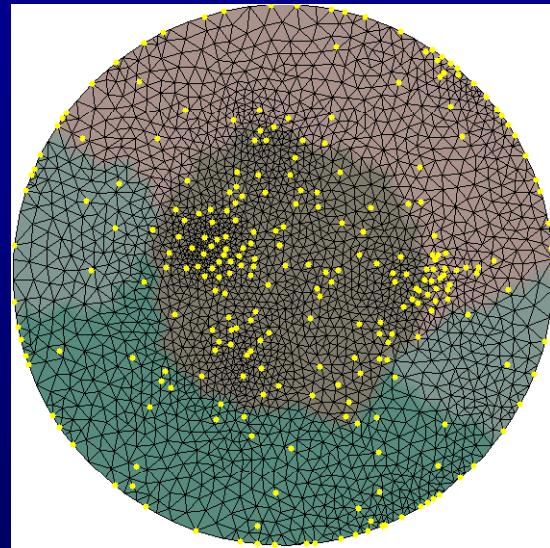
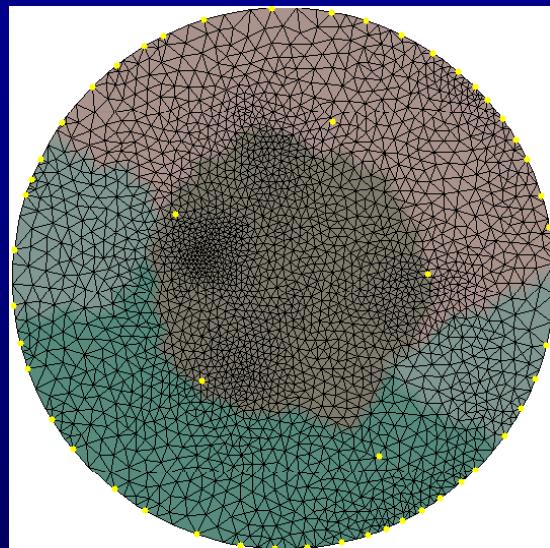
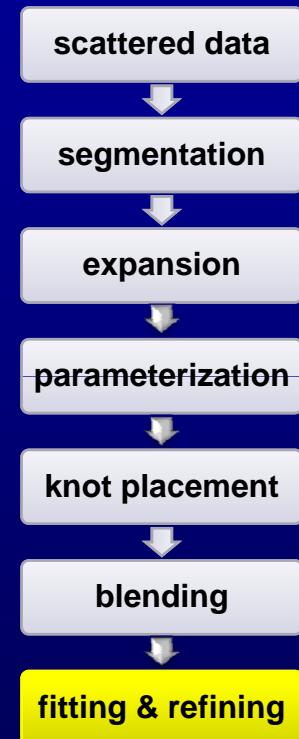


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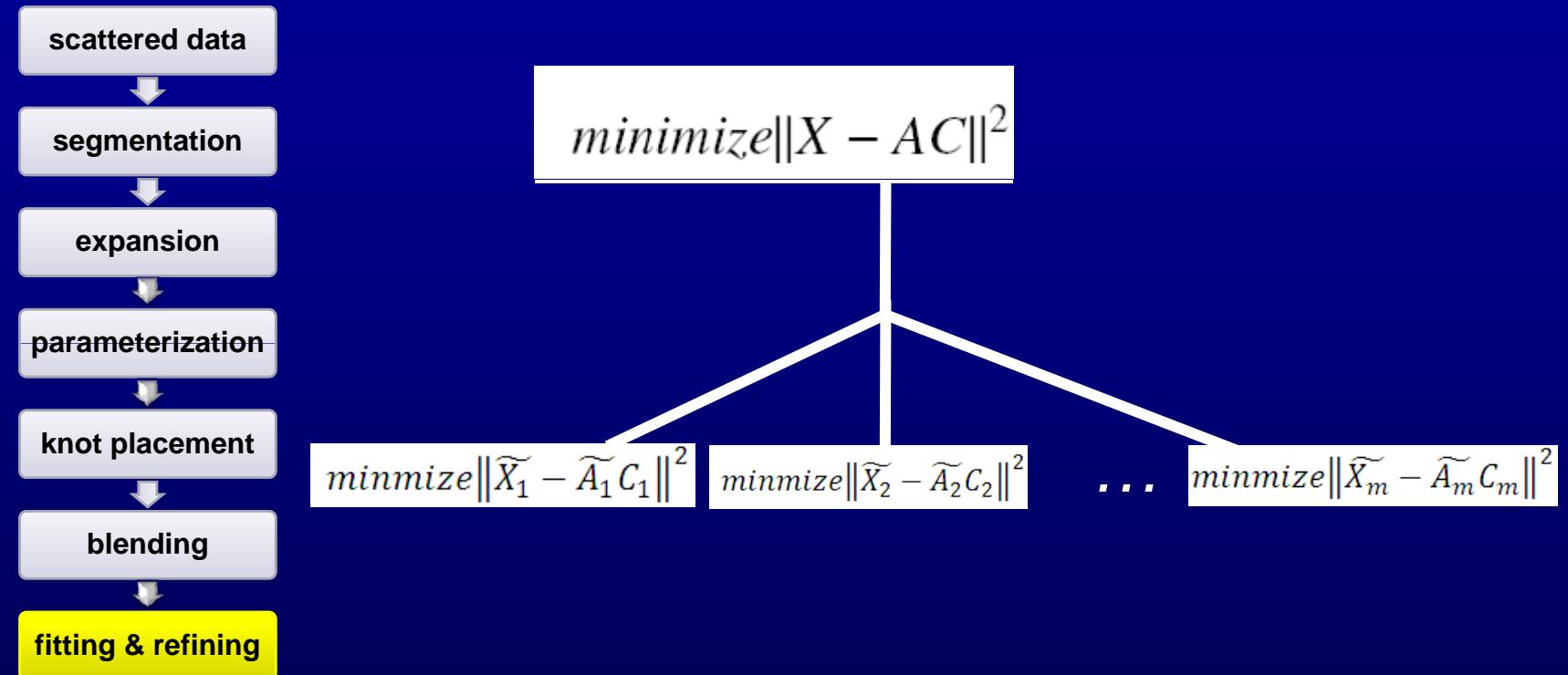
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Performance

<i>Model</i>	<i>Nv</i>	<i>Nc</i>	<i>Nd</i>	<i>m.e.</i>	<i>rms</i>	<i>it-time</i>
<i>Gargoyle</i>	59k	9k	12	0.34%	0.049%	51.9
<i>Rocker-arm</i>	50k	13k	15	0.11%	0.012%	74.8
<i>Igea</i>	50k	12k	20	0.44%	0.024%	70.3
<i>Kitten</i>	33k	7k	17	0.50%	0.027%	30.0

The statistics of scattered datasets and surface fitting configurations. Nv: number of vertex; Nc: number of control points; Nd: number of domain; m.e.: maximum error; rms: root-mean-square error; it-time, the running time for one global fitting iteration.

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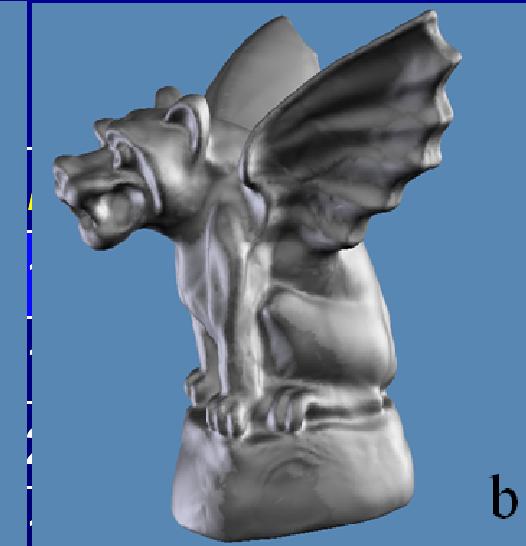
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Performance

<i>Model</i>
<i>Gargoyle</i>
<i>Rocker-arm</i>
<i>Igea</i>
<i>Kitten</i>



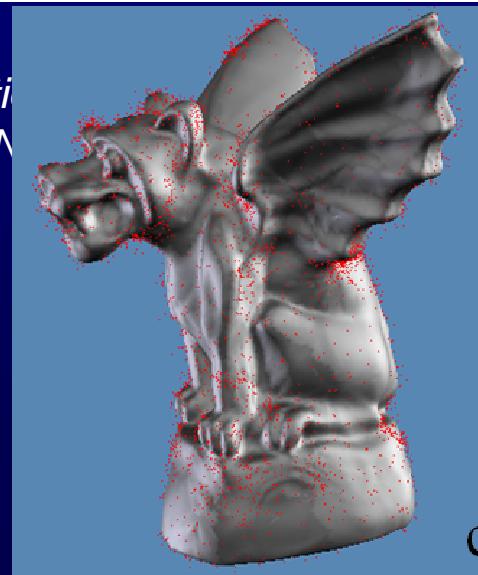
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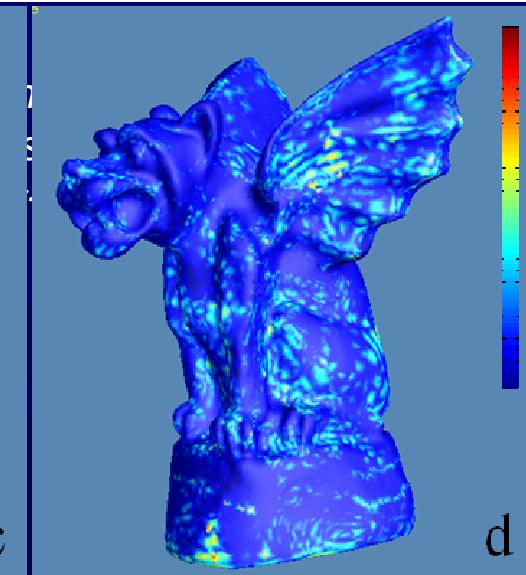
b

<i>s</i>	<i>it-time</i>
49%	51.9
12%	74.8
24%	70.3
27%	30.0

The statistics:
of vertex; Nv:
error; rms:
iteration.



c



d

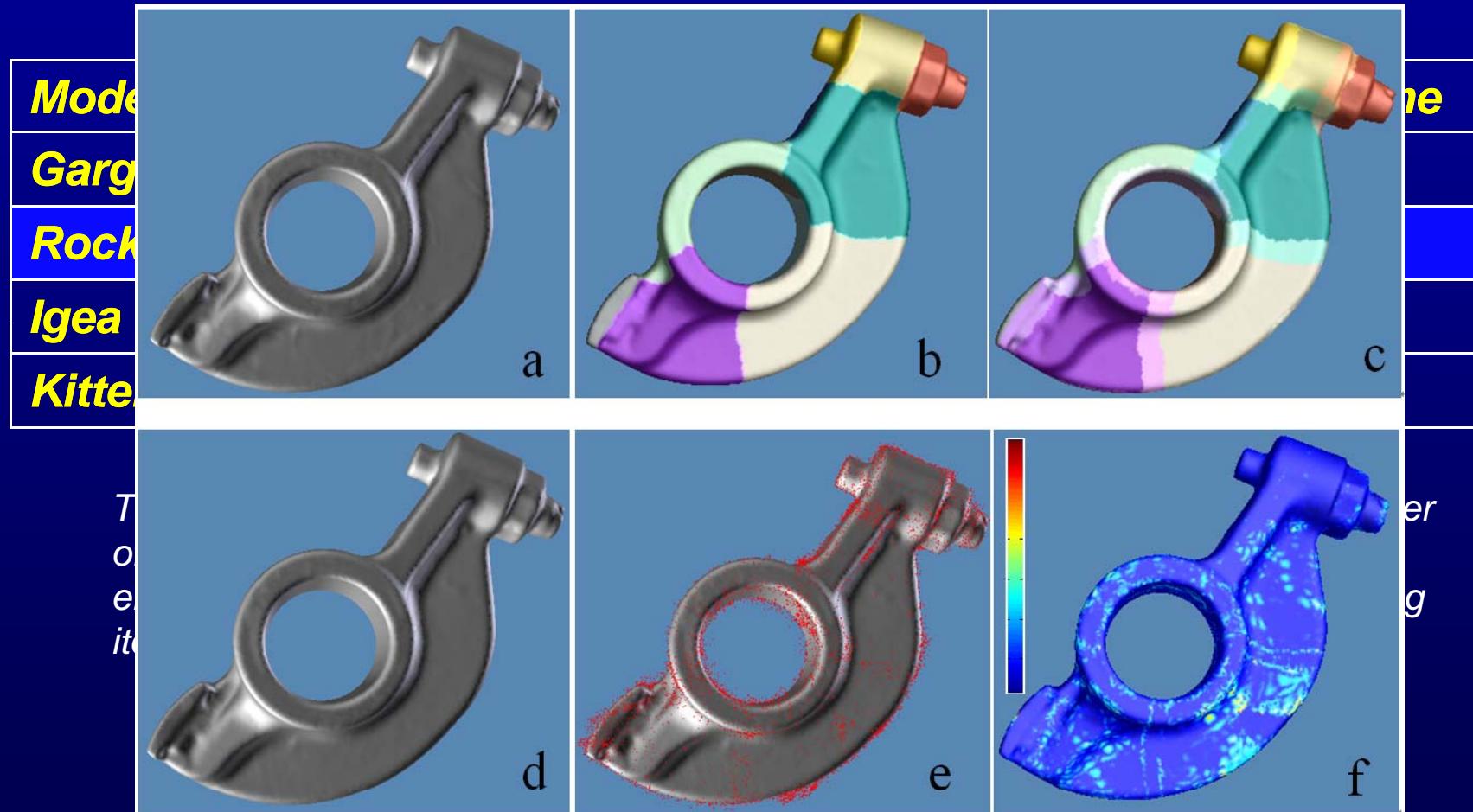
ons. Nv: number
i.e.: maximum
one global fitting

Performance

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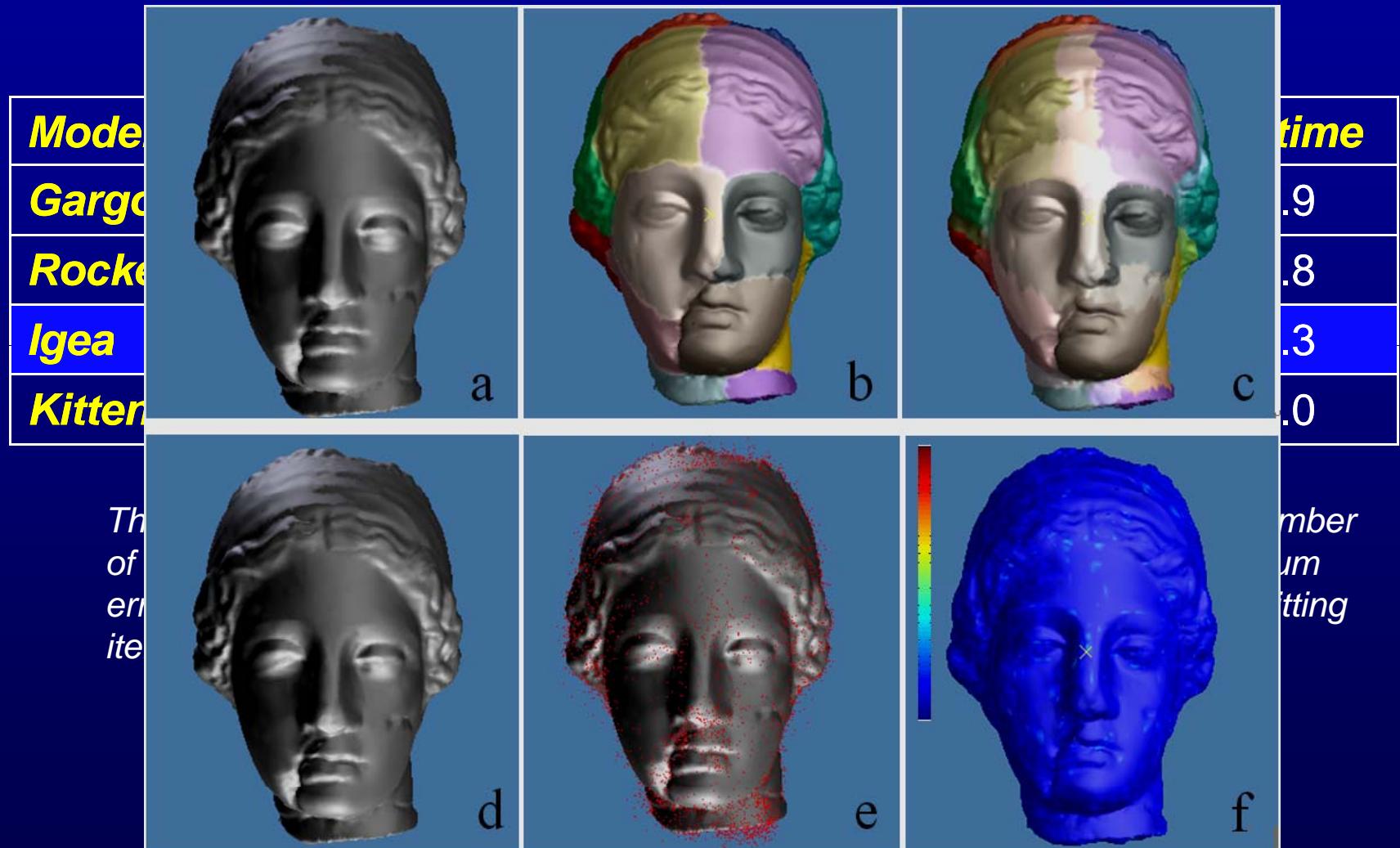


Performance

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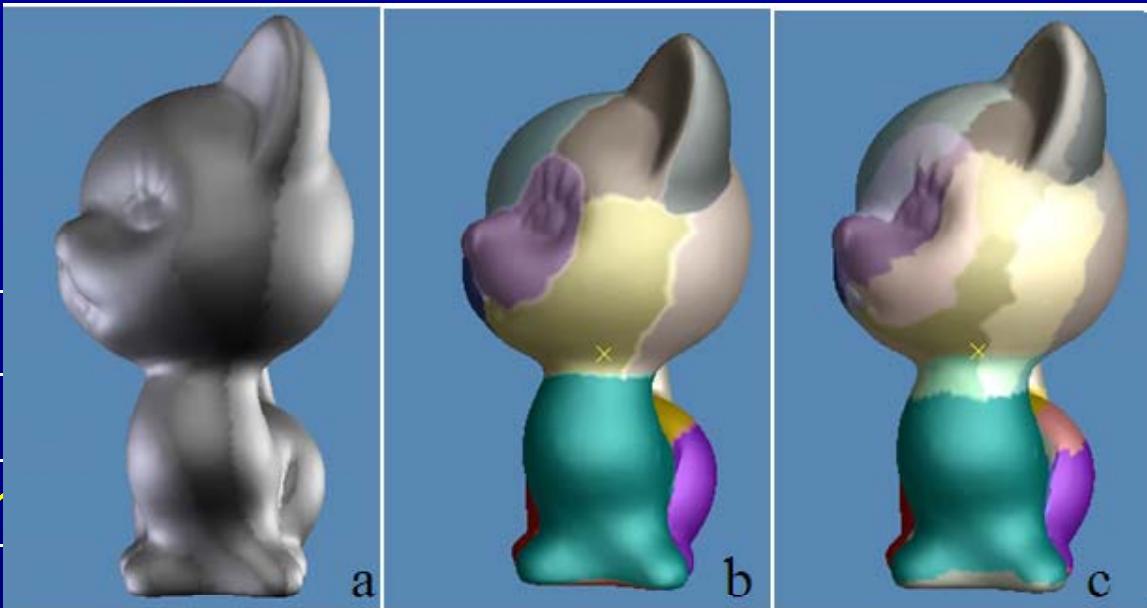


Performance

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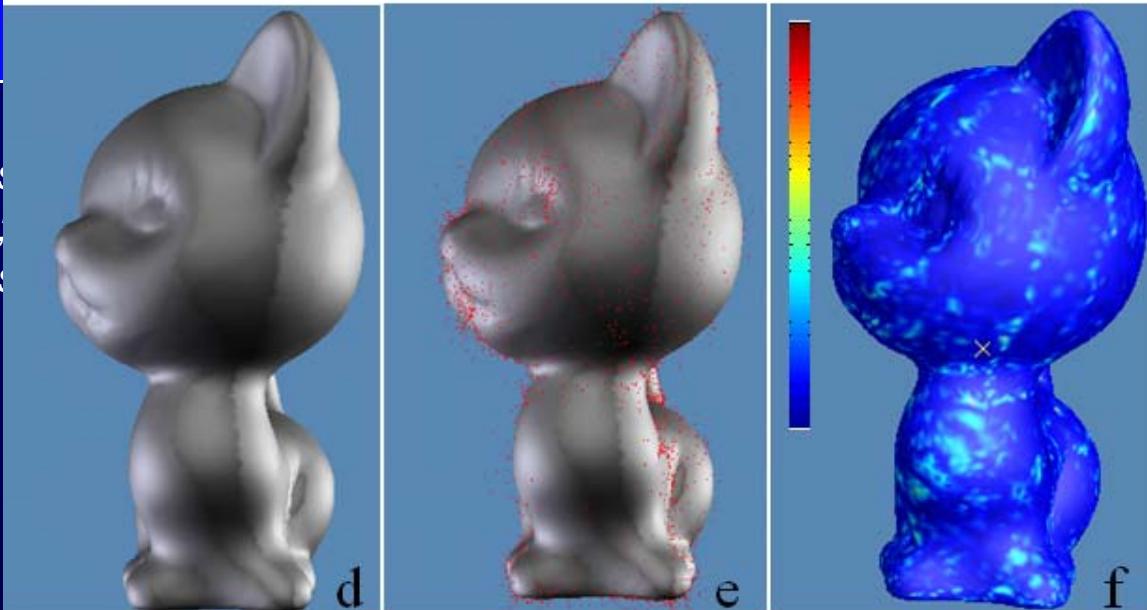
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Model
Gargoyle
Rocker-arrow
Igea
Kitten



it-time
51.9
74.8
70.3
30.0

The statistics of vertex error; rmse: root mean square error; it: iteration.



lv: number of maximum global fitting

Summary

- A new type of freeform parametric surfaces for shape modeling
- Handle triangle mesh with arbitrary topology
- Completely avoid auxiliary knots
- Adaptive fitting process for surface refinement

Future Research Directions

- Handle open meshes
- Improve surface fitting quality and surface blending schemes
- More flexible knot placement methods
- Generalization to trivariate volumetric splines
- Generalization to more practical applications beyond surface reconstruction

Acknowledgements

- National Science Foundation: IIS-0830183,
IIS-0710819

Thank You!